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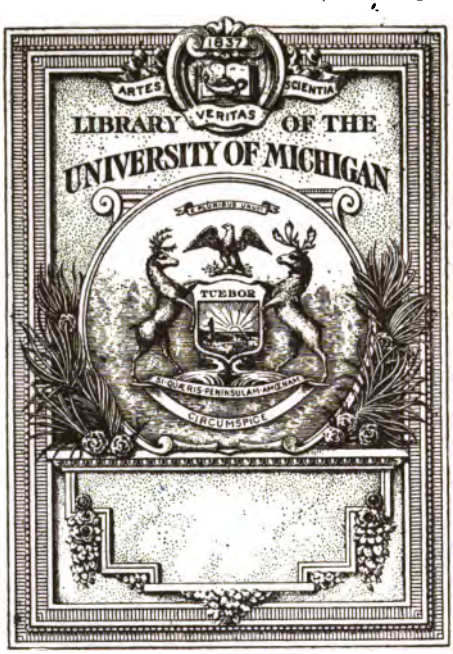
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SIMPLIFIED NAVIGATION
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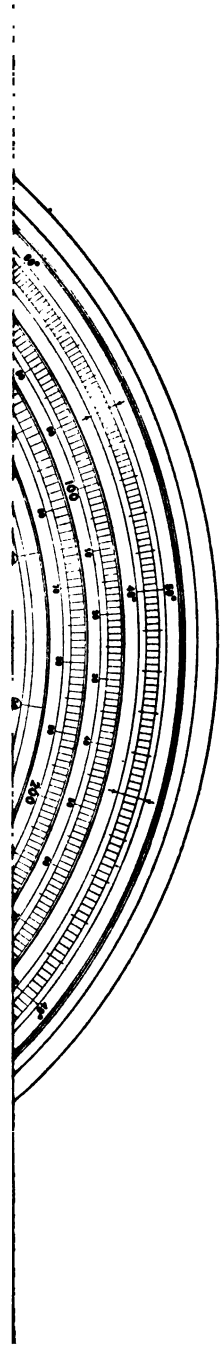

SIMPLIFIED NAVIGATION
FOR
SHIPS AND AIRCRAFT

A TEXT BOOK BASED UPON
THE SAINT HILAIRE METHOD

BY

CHAS. LANE POOR

Professor of Celestial Mechanics in Columbia University
Author of "The Solar System," "Nautical Science,"
"Sumner Line Tables Simplified," etc.

ILLUSTRATED

NEW YORK
THE CENTURY CO.
1918

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Published, September, 1918

Rec. 11-10-31 MEIN

FOREWORD

BY

REAR ADMIRAL BRADLEY A. FISKE, U. S. N.
(Retired)

THE sciences of mathematics and astronomy are the sciences that govern the navigation of the sea. Their application to navigation was partially understood and crudely practised, before Columbus started across unknown waters, to discover the way to India in 1492. A continuous progress in that application has since been going on, at a continuously increasing speed, and it has made possible that movement of vessels over all the oceans, without which the present civilization of the world could not exist. This book is the first serious attempt to extend the application of those sciences in a definite and practical way to the problem of the navigation of the air.

The early mariners propelled their little craft by oars and afterwards by sails over short distances, and not far away from the safe shelter of the land. Trips of canoes and rafts and boats were undertaken, there is reason to sup-

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pose, in far away times, and in all places that bordered on oceans, lakes and rivers, but the best accounts we have are of voyages along the eastern shores of the Mediterranean. As time went on, and the mechanic arts developed, mariners were able to get larger and stronger craft, and to push out farther and farther from the shore; but it was not until the invention of the mariner's compass that they had any guide as to how to direct their course, except such as was given vaguely and approximately by the sun, moon and stars, and more clearly by marks on shore which they could see. Later, the sextant was invented. By this instrument the height of a heavenly body above the horizontal could be measured. This height being known, and certain mathematical and astronomical formulæ being then applied, the locality of the vessel wherefrom the height was measured could be computed.

The advance which has taken place, not so much in the science of navigation as in its practice, during the last 100 years and especially during the last 20 years, has been enormous. This advance was brought about mainly by two things; first, the necessity for such an advance by reason of the greater draft of ships, their

greater speed, and the commercial importance of being able to make trips on pre-arranged schedules; and second, the great advance of the mechanic arts, especially in the invention and development of methods and instruments of precision.

Even in Nelson's day, 100 years ago, ships were of such small draft and moved so slowly and in such uncertain directions, because of the uncertainty of the propelling element—the wind —and navigational appliances were so crude and clumsy, that quick and accurate navigation was not only unnecessary but impossible. But at the present day, when instruments can be constructed whose parts are correctly made to within one ten thousandth of an inch, when steamers of 20,000 tons displacement (and more) charge across the ocean in all directions at speeds of 20 knots per hour (and more) and enter and leave ports on schedule time, going through channels having depths barely sufficient to float the ships; and when fleets of battleships, battlecruisers, destroyers and other craft maneuver together, even at night with no lights showing outside, quickness and accuracy in sea navigation are not only necessary but possible; and to a degree that Nelson never dreamed of.

Before the art of navigation on the sea was born, the art of navigation on the land, or the art of moving over the land in desired directions and over desired distances, had attained a considerable development. The art of the navigation of the sea coming later, as a product of a higher civilization, has become a more scientific and difficult art than was its predecessor. It has become an exceedingly scientific and difficult art; and now we see the coming of a still later art, which will be still more scientific and difficult—the navigation of the air.

At the present time, this art is in its infancy and Professor Poor is the pioneer. From one point of view, the navigation of the air is easier than the navigation of the sea, for the reason that less accuracy is required; since an airplane does not depend for its safety on such exact determinations of its position relative to the land. This is because, while the same heavenly bodies guide an airplane as guide a ship, and while the same observations of their height and the same resulting computations can be employed, an error of a mile may run a ship ashore and wreck it, while an error of even ten miles would not cause a like disaster to an airplane.

From another point of view, however, the navi-

gation of the air is the more difficult art; because it is much harder to handle instruments and make computations in the confined spaces of an airplane, rushing along 100 miles an hour, than it is to handle instruments and to make computations in a comparatively commodious and slow-moving ship. In order to overcome the difficulties presented by airplane conditions, Professor Poor has not only simplified certain navigational methods, but has also invented and constructed a new navigational instrument.

As time goes on, the navigation of the air will doubtless follow the same course as other civilized arts have followed, and become increasingly scientific and correct; obeying, as the art of sea navigation did, the laws of necessity and opportunity. Reason and history tell us what the future has in store; but the imagination fails to picture it definitely, and only vague visions flash before the mind of air squadrons moving at high speed over all the oceans and countries of the world; alighting, sometimes by day and sometimes by night, at pre-determined points on sea and land; and rising from those points again, to go to other points;—with the same confidence as that with which the captain of a ship now begins a trip from Norfolk to New York, or the con-

ductor of a train starts from Washington to Baltimore.

The limitations of men's minds are such that we can never proceed from any stage of development to another stage, except by a long series of intervening steps. The first thing to do is to make the first step. The first long step in aerial navigation that is not a mere stumbling in the dark, has been made by Professor Charles Lane Poor, and explained by him in this book. The eminent position of Professor Poor as a mathematician and astronomer is a guaranty of the excellence of his work; and the imminence of the coming of a world-wide utilization of aeronautics is a guaranty that the first step of Professor Poor will be followed by countless successive steps as time goes on, towards the realization of man's paramount present hope—the conquest of the air.

B. A. F.

PREFACE

THIS work is intended to assist in the training of the thousands of navigators, who will, in the future, man our ships on the sea and in the air. It is an attempt to explain in non-technical language and without the use of complicated mathematical formulas the principles that form the basis of modern navigational methods.

The Sumner method of finding one's position is fundamental, and when combined with the modern methods of reduction, first introduced by Admiral St. Hilaire, is most simple and is readily applied to all navigational problems. It may be used with observations made at any time, with equally good and consistent results: it is applicable to all conditions and is available regardless of azimuth, altitude, or hour angle. The time-honored noon sight for latitude and the morning or afternoon sight for longitude are but special cases of this most powerful method. The St. Hilaire method provides one simple, straightforward, standard method of reducing all sights

and embodies everything essential to safe navigation.

New problems are introduced by aerial navigation, but, at the same time, many of the difficulties of marine navigation are largely eliminated. In a plane, flying at one hundred miles an hour, the navigator cannot spend many minutes in making long and elaborate calculations, but on the other hand he has no rocks nor shoals to fear, and he need not determine his position with the exactitude required at sea. Simplicity and speed in working a sight are essential; minute accuracy is non-essential.

C. L. P.

August, 1918

CONTENTS

		PAGE
	FOREWORD	vii
	PREFACE	xiii
CHAPTER I	FINDING ONE'S POSITION AT SEA AND IN THE AIR	3
II	DETERMINATION OF THE "D.R." ALTITUDE AND AZIMUTH	23
III	CORRECTIONS TO BE APPLIED TO THE MEASURED ALTITUDE	72
IV	SIMULTANEOUS ALTITUDES	89
V	NOTES AND PRACTICAL APPLICATIONS, WITH SPECIAL REFERENCE TO AERIAL NAVIGATION	99

TABLES

TABLE		PAGE
I	Dip and Distance of the Horizon for the Average State of the Atmosphere . .	113
II	Correction for Refraction, Parallax, and Semi-Diameter to be Applied to the Observed Altitude of the Sun's Lower Limb, and the Correction for Refraction to be Applied to the Observed Altitude of a Star (Marine Table)	115
III	Correction for Refraction, Parallax, Semi-Diameter, and Dip to be Applied to the Observed Altitude of the Sun's Lower Limb, and the Correction for Refraction and Dip to be Applied to the Observed Altitude of a Star (Aerial Table)	119
IV	Fifteen Useful Navigational Stars . .	125

ILLUSTRATIONS

FIG.		PAGE
	The Line of Position Computer .	*Frontispiece*
1	A Circle of Position	12
2	A Sumner Line of Position	18
3	Finding the Altitude	26
4	A Sumner Line and Aquino's Assumed Position	39
5	Position 1. Setting for Declination and Latitude	56
6	Position 2. Setting for Hour Angle . .	58
7	Position 3. Setting for Difference Between Latitude and Declination	59
8	Position 4. Finding the Altitude . . .	60
9	Position 1. Setting for Hour Angle and Altitude	61
10	Position 2. Finding the Azimuth . . .	62
11	Comparison Between the Sumner Lines as Found by the Logarithmic, the Aquino, and the Mechanical Methods	66
12	Refraction	75
13	Dip of the Horizon	77
14	Simultaneous Altitudes	90
15	Lines of Position	93
16	Finding the Intersection of Lines of Position	95

SIMPLIFIED NAVIGATION

SIMPLIFIED NAVIGATION

CHAPTER I

FINDING ONE'S POSITION AT SEA AND IN THE AIR

ASTRONOMICAL observations furnish the only means of finding one's position at sea, upon unknown and unmapped portions of the land, or in the air, when out of sight of familiar and easily recognized objects. For short coasting trips from port to port, the navigator may depend upon pilotage, and the compass and the log may suffice. But the oceans are full of streams and currents, whose directions and speeds change from day to day, and dead reckoning is not to be relied upon. Cases are on record where vessels have been carried eighty miles off their courses in a single twenty-four hours. In modern steel and iron vessels the compass is subject to deviations, and unless carefully tested and adjusted from time to time it is not to be trusted; and too

implicit a reliance upon the compass has been the cause of many a shipwreck.

Aerial pilotage has been most highly developed, and as long as the aviator is in sight of well-mapped portions of the land he can guide his plane from point to point with ease and certainty. On account of the great velocities of the winds, which may be blowing in any direction relative to the course of the plane, the sidewise drift of the machine may be very great. Drifts of twenty and even thirty degrees are not uncommon. The compass alone, under such circumstances, is useless. By the use of the automatic drift set, however, in connection with the compass, this drift of the plane can be compensated and the machine kept on a direct compass course. This instrument, however, can be used to advantage only when the plane is over land and when the land formation is such that the "streamlines" are clearly visible. For short trips over the surface of the ocean, when the direction of the wind remains constant, this instrument is of value and the aviator is enabled to hold the proper course. But for long and high flights,

and especially for transoceanic flights, the drift becomes uncertain, and the aviator may wander hundreds of miles from his proper course. To locate oneself under these circumstances recourse must be had to astronomical observations.

The principles upon which is based the art of finding one's position on or over the surface of the earth, are very simple. The heavens appear like a great hollow globe surrounding the earth: the sun, the planets, and the stars, each marks a definite point on this celestial globe. The two spheres, the earth and the heavens, correspond to one another, point to point and line to line. To the north pole of the earth corresponds the north pole of the heavens; to the equator of the earth, the celestial equator; the north pole of the heavens, closely marked by the pole star, is directly over the north pole of the earth; the celestial equator is directly over the earth's equator. When, now, we want to locate the position of a town or city on the earth, we give its latitude and longitude; its distance north or south of the equator, and its distance east or west from Greenwich. Similarly in the heavens, the position of each star

is known by its distance from the equator, its declination, as it is called, and by its distance from the celestial Greenwich, or its right ascension. Declination corresponds to latitude; right ascension corresponds to longitude, excepting that right ascension is measured always to the east, while longitude is measured both to the west and to the east. If the celestial first meridian were directly overhead at London, then the latitude and longitude of any and every point on the earth's surface would correspond exactly to the declination and right ascension (except for method of measurement) of that point of the heavens which was in its zenith. Directly over New York would be found a small star barely visible to the sharpest eye, whilst the brilliant star Vega would mark the place of the little town of Redknob in West Virginia.

If the heavens were thus fixed and a traveller on the earth's surface were provided with a chart of the heavens, he could, by finding and identifying a star directly overhead or in his zenith, determine his latitude and longitude, for his latitude and longitude would be the same as the declina-

tion and right ascension of the star. It is not necessary for the traveller to find a star exactly in his zenith, provided its distance from the zenith can be accurately measured. If, for example, Vega be directly over the town of Redknob, and a traveller find Vega to be one degree (1°) south of his zenith, then he knows that he is one degree, or sixty geographical miles, north of Redknob, or in the immediate vicinity of Caldwell, Ohio. The traveller will always be just as many miles to the north and west of Redknob as Vega appears minutes of arc south and east of his zenith; and, vice versa, when he is east and south of Redknob, Vega will appear north and west.

This is the whole essential of navigation,—a star of known position in the heavens is selected and its distance from the observer's zenith measured. The traveller will then be as many miles from the point on the earth's surface directly under the star, as the star is minutes of arc from the zenith. To determine the observer's direction from the sub-stellar point it is only necessary to note the azimuth of the star. The distance of the star gives distance, the azimuth gives direc-

tion. The zenith distance is measured with a sextant, the azimuth may be roughly found by compass, or more accurately determined by calculation.

So far it has been assumed that the heavens appear stationary, that over each point of the earth's surface is a fixed star. This is not so: the earth rotates on its axis once in twenty-four hours and causes all the heavenly bodies apparently to describe small circles about the pole, rising in the east, passing across the heavens, and setting in the west. Each star thus travels over its own circle of latitude; Vega, for example, will, at some instant of each day, appear directly in the zenith of every point of the earth's surface, whose latitude is the same as Redknob. This rotation of the earth is perfectly uniform and furnishes our measure of time; clocks, sidereal or solar, are regulated and governed by this rotation. Sidereal clocks indicate noon, or zero hours, when the first meridian of the heavens is directly overhead; one o'clock when this prime meridian has passed one twenty-fourth of the circumference to the westward. Vega, whose

right ascension is 18^h 33.8^m, will always cross one's meridian when the clock points to 18^h 33.8^m. Three hours before this time, or at 15^h 33.8^m, Vega will be over a place situated three hours, or 45°, east of the meridian for which the clock is set. Thus by merely noting the sidereal time, the exact spot on the earth's surface in the zenith of which Vega appears is at once known. To find the sub-stellar point, therefore, it is necessary for the observer to know the sidereal time at which the observation is made. He must have a clock or chronometer regulated to some standard meridian, from which the time can be taken.

Exactly similar reasoning applies to the case of the sun. At apparent noon at Greenwich the sun is in the zenith of that point on the Greenwich meridian whose latitude is equal to the sun's declination. As the earth rotates, the sun (except for its variation in declination) passes along a circle of latitude on the earth, and each hour appears on the zenith of a place 15° farther to the westward. At five hours Greenwich apparent time the sun will be in the zenith of a place 75° to the westward of Greenwich. At any instant,

this circle, whether on land, on sea, or in the air, the sun would appear 40° from his zenith, or 50° above his horizon. The observed altitude by itself, therefore, would merely indicate to the observer that he was at some point on this circle,—neither inside nor outside of it, but on it; and

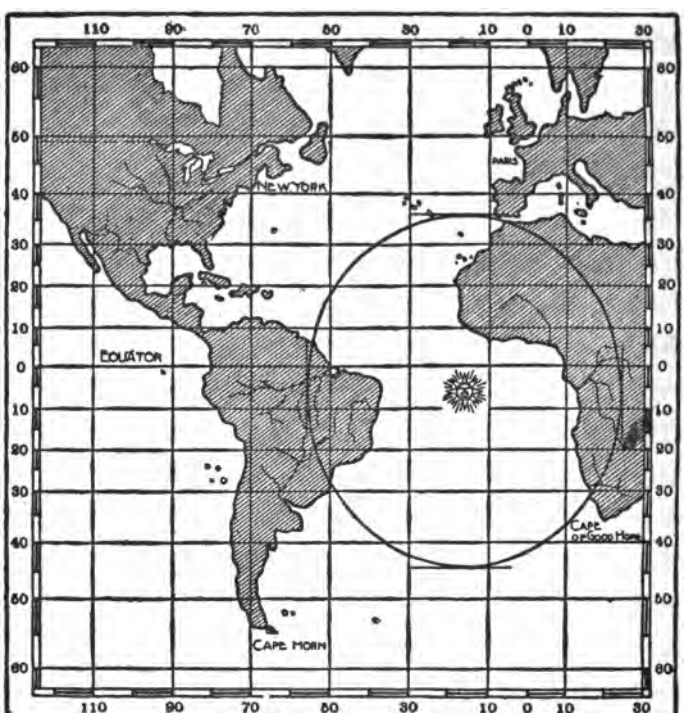


Fig. 1. A Circle of Position

this is the sum total that can be derived from the measured altitude. Mathematical calculation is no use, logarithms are of no avail, the measured altitude tells the observer that he is on this circle, but not upon any definite portion of it. To find his exact position on the circumference of this circle, the navigator must make some other observation, or have some further data to aid him.

In practice the problem is not quite so indeterminate as it would appear to be, for the traveller has some idea as to his location and knows, in a general way, on what part of the circle he really is. Except in very unusual cases he knows his position within a degree or so, and thus only a small portion of the circle is necessary. Now in general the circle is very large, and any small portion of its circumference may be considered a straight line. Such a straight line is in reality a tangent to the circle and coincides with it for a short distance, and the traveller knows, as the result of his observation, that he is in fact situated at some point of this straight line. The "circle of position" becomes a "line of position," and such a line is known as a "Sumner line," after

Capt. Thos. H. Sumner of Boston, who, in December, 1837, discovered the basic principle of modern navigation. Perhaps no better explanation of the practical application of the Sumner line can be made than that given by Captain Sumner himself. In his book, published in 1843, after describing a very stormy passage from Charleston, South Carolina, to Greenock, when for many days observations were impossible, he continues:

". . . At about 10 A. M. (18th December) an altitude of the sun was observed, and chronometer time noted; but having run so far without any observation, it was evident that the latitude by dead reckoning was liable to error and could not be entirely relied upon.

"However, the longtitude by chronometer was determined,—using the uncertain D. R. latitude, and the ship's position fixed in accordance. A second latitude was then assumed 10′ to the north of the last, and working with this latitude a second position of the ship was obtained; and again a third position by means of a third latitude still 10′ further north.

"On pricking off these three positions on the chart it was discovered that the three points were all disposed in a straight line lying E.N.E. and W.S.W., and that when this line was produced in the first named direction, it also passed through the Smalls light. The conclusion arrived at was, 'that the observed altitude must

have happened at all three points, at the Smalls light, and at the ship at the same instant of time.' The deduction followed that, though the absolute position of the ship was doubtful, yet the true bearing of the Smalls light was certain, provided the chronometer was correct. The ship was therefore kept on her course, E.N.E., and in less than an hour the Smalls lights was made bearing E. by N. ½ N. and close aboard. The latitude by D. R. turned out to be 8′ in error."

In this, the first time a Sumner line was used, its essential principles as a "line of position" are clearly stated. The measured altitude of the sun, or star, determines a line upon which the observer must be situated. His exact location on the line is unknown, but the position and direction of the line itself are fully and completely determined.

While in theory it is easy to describe a "circle" or "line of position," yet in actual practice it has been found rather difficult actually to locate and draw on a chart the "Sumner" line of position. The mathematical formulas known to Captain Sumner were cumbersome, and the tables then available were unsuited to the problem, and the labor of working a "sight" was almost prohibitive. Since then, however, many simplifications

have been introduced; special tables computed, and the method perfected. But, at its best, the original Sumner method as given in the latest treatises on navigation requires considerable calculation and it fails when the observed body is near the meridian. A few years ago Admiral Marcq St. Hilaire, of the French Navy, developed, under the name of "new navigation," a modification of the Sumner method, which is theoretically more perfect, being equally accurate whether the observation be made on the prime vertical or near the meridian, and which greatly simplifies the calculations necessary to working a sight.

To apply this method, the traveller by sea or air lays down on the chart the position in which he thinks he is at a given time. This assumed position is calculated from the courses travelled and speeds made since leaving port, or the last known station. This is known as the position by dead reckoning, or briefly as the "D. R." position. From this position an observer would, at a given instant, see the sun at a certain definite altitude, and this altitude can be calculated.

This calculated altitude has nothing to do with any actual observed altitude; it is purely theoretical, and shows at what altitude the sun would appear were the traveller actually in the place he thinks he is. It may be called the "D. R." altitude.

At the instant for which he has made his calculation, the traveller measures with his sextant the altitude of the real sun, and this measured altitude defines a line of position on which he must be. In general, this line will not pass through the "D. R." position of the traveller as plotted on the chart. Only when the traveller's position by dead reckoning agrees exactly with his true position, will the Sumner line pass through the "D. R." position. The assumed, or charted, position will usually appear to be on one side or the other of the Sumner line, or at O of the accompanying diagram. If the measured altitude be greater than the calculated, or "D. R." altitude, then will the line of position be between the subsolar point and the "D. R." position, or the traveller will be nearer the subsolar point than he thought he was. On the other hand, if the meas-

ured altitude be the smaller, then will the line of position be further from the subsolar point. In either case the distance, measured in nautical, or geographical miles, between the Sumner line and the "D. R." position will be equal to the difference, in minutes of arc, between the observed and the calculated altitudes. This distance is measured along the line connecting the "D. R." position with the subsolar point, and the direction in which this line lies on the surface of the earth, is given by the azimuth of the sun.

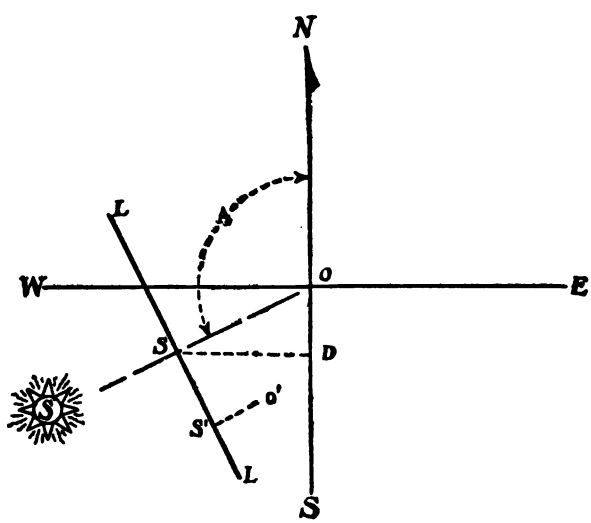


Fig. 9. A Sumner Line of Position

To pass, therefore, from the assumed, or "D. R." position to the Sumner line on which the traveller really is, two factors must be determined: a distance and a direction. The first of these, the distance, OS, of our diagram, must be found with accuracy. An error of one minute of arc (1′) in either the calculated or the observed altitude will cause a corresponding error of one mile in determining the position of the Sumner line. On the other hand, the azimuth need be found only approximately; an error of one degree (1°) in the bearing will introduce an error of only about one-half (½) mile at the ends of a Sumner line sixty (60) miles long. Such an error is insignificant and practically negligible. The bearing, or azimuth, of the sun can, therefore, be taken with sufficient accuracy from any one of a number of Azimuth Tables, or can be roughly calculated at the same time the "D. R." altitude is computed.

The distance factor, OS, is the difference between two altitudes; the one observed, the other calculated, and each of these must be determined with accuracy. The methods by which these alti-

tudes are determined with the necessary accuracy are fully discussed in following chapters.

The Sumner line may be plotted on the chart, and the most probable position of the ship, S, found. Or the latitude and longitude of this point may be found directly from the "D. R." position by a very simple calculation. The line, OS, of the diagram may be considered as a course sailed by the vessel, and the corresponding difference of latitude, OD, and the departure, DS, can be found at once from the Traverse Table, using OS as the distance and the azimuth as the course. After converting the departure into difference of longitude, the differences of latitude and longitude, thus found, can be applied directly to the "D. R." position, and hence the latitude and longitude of S found, without drawing the Sumner line and without reference to the chart. While this calculation is very simple, it is generally better to plot the Sumner line on the chart, for the actual seeing of the line is a great help. The line may often give all the information desired, may show the distance of the ship from the

shore, or may indicate the course to be sailed in order to make a good land-fall.

It must always be remembered that S is only the most probable position of the vessel, the actual location may be at any point of the Sumner line, LL. If, instead of the "D. R." position, O, some other position O′ had been assumed as the location of the vessel and the calculations made accordingly, then different values of the distance factor and the azimuth would have been obtained. One would have found, under these conditions, O′S′ for the difference of the altitudes and for the azimuth a value slightly different from the former. Plotting these values on the chart, however, leads one to a Sumner line, LL, identical with the one from the "D. R." position. Starting with O′, however, the most probable position of the ship would be at S′. Thus, so far as finding the Sumner line itself is concerned, the exact location of the starting point, or assumed position of the ship, is a matter of indifference; any point, within reasonable limits, can be selected. The "D. R." position is, of course, the most natural position to take, as it is the one in which

the navigator believes himself to be. While the Sumner line is thus fully determinate, the point, S, is not, for the location of S depends upon the assumed location of the vessel.

If the observation should be made with the sun bearing due east or due west, then the Sumner line would run north and south and the observation would fully determine the longitude of the observer. Such an observation corresponds to the old-fashioned morning or afternoon longitude sight. If the observation be made at noon, with the sun bearing south, then the Sumner line runs east and west, and the observation fully determines the latitude of the observer. This corresponds to the ordinary noon sight for latitude. Now, the great advantage of the St. Hilaire method is that all sights are handled in the same way, and that equally accurate results are obtained no matter what the azimuth of the sun may be. By this method the whole of navigation is reduced to a standard form; every sight is worked out by the same formula and in identically the same way.

CHAPTER II

DETERMINATION OF THE "D. R." ALTITUDE AND AZIMUTH

THE "D. R." altitude, or the altitude of the sun as it would be seen from the place in which the traveller supposes himself to be, can be found by mathematical calculation only. This calculation involves the solution of a spherical triangle, and this can be effected only by the formulas of trigonometry. By the aid of special tables, or mechanical devices, however, these calculations may be put into such simple form that the practical navigator need have no knowledge of trigonometry and can solve the problem with a few simple additions.

The navigator begins with his "D. R." position, that is, with the latitude and the longitude of the place in which he believes himself to be, together with the chronometer time at which he

made an observation of the sun. The chronometer time is first corrected for "chronometer error," and then to this result is applied the "equation of time," as taken from the "Nautical Almanac," and thus is found the Greenwich Apparent time at which the observation was made. From this subtract the longitude of the place, if west (expressed in hours); add, if east; and the result is the Local Apparent time, or ship's time.

This measures the angular distance of the sun east or west of the meridian, which angular distance is known as the Hour Angle. On ordinary clocks or chronometers, however, the noon hour is denoted by twelve (XII), so that an apparent time of eleven (XI) hours means that it will require one hour for the sun to reach the meridian. Hence, in the morning the hour angle is found by subtracting the apparent, or ship's time, from twelve (12) hours, but in the afternoon the apparent time and the hour angle are identically the same. The hour angle, thus found, will hereafter be denoted by t.

From the "Almanac" the navigator also finds the declination of the sun for the moment the

observation was made, and this declination is denoted by D. This is the third and last of the known quantities which the navigator uses in his calculations. These quantities are:

L, the latitude by dead reckoning.
D, the declination of the sun.
t, the apparent time, or the hour angle of the sun.

The essential elements of the problem are shown on the following diagram, which represents the earth's hemisphere as viewed from that point over the equator whose longitude is equal to the longitude of the subsolar point, S. The "D. R." position of the observer is at Z; EE represents the equator and P the north pole. The "D. R." latitude of the observer is represented by LZ, the declination of the sun by ES, and the sun's hour angle, or the apparent time, by t, the angle ZPS. The zenith distance of the sun is given by SZ, and hence the required altitude by HS.

In the spherical triangle ZPS the three sides are functions of the latitude, the declination, and

the altitude, being respectively 90°–L, 90°–D, and 90°–h. Spherical trigonometry furnishes a definite relation between these three quantities and the hour angle, t, and this relation is expressed by the formula

$$\sin h = \sin L \sin D + \cos L \cos D \cos t,$$

and this formula is the fundamental formula upon which the whole of navigation rests. It is

Fig. 3. Finding the Altitude

used for many purposes and in many different ways. It involves four quantities; if any three of them are known, the fourth can be found by calculation. It can be transformed and put into better and more convenient shape for numerical calculation, the exact shape in which it is used depending upon the unknown quantity sought.

To a practised computer this formula is very simple and easy of application, but to the navigator it is rather complicated and many attempts, therefore, have been made to simplify it. Some of these methods are logarithmic and give accurate results, others are approximations merely, and give only a rough and ready solution. These various methods may be divided into three general classes as follows:

a. Solutions by Logarithm Tables

These are the direct mathematical methods, are the most accurate, and are the only ones available when great precision is required.

b. *Solutions by Double Entry Tables*

These tables were first made by Sir William Thomson (Lord Kevlin) and afterwards perfected by

de Aquino. These tables are difficult to use, as they require double interpolations.

c. *Mechanical Solutions*

By means of mechanical devices very rapid but approximate solutions may be obtained. For purposes of "off shore" navigation, or for air-plane work, these devices are satisfactory, but they cannot be used where extreme accuracy is essential.

These three methods are fully developed in the following sections.

ALTITUDE AND AZIMUTH

a. *Solutions by Logarithm Tables*

The simplest and quickest way of solving the problem, as actually presented in practice, and with the required degree of accuracy, is by means of the Cosine-Haversine method, with the aid of specially arranged logarithmic tables. The ordinary tables of logarithms, as furnished the navigator, are inconvenient and badly arranged. To a large extent this is due to the fact that the tables have been arranged for the solution of any and all problems that might possibly arise in navigation, or in astronomical research; they contain a large amount of matter that is totally unnecessary to the working out of a sight. Tables, arranged to facilitate the work as much as possible and to put the computations into the simplest and readiest form, have been published by T. S. & J. D. Negus, under the name of "Sumner Line Tables, Simplified," and by the U. S. Hydrographic Office, under the name of "Line of Position Tables." Both sets of tables are used in the same way and the following explanation,

while directly applicable to the Sumner Line Tables, may be used with minor modifications for the Line of Position Tables.

To use these tables the fundamental formula is transformed by simple trigonometrical relations into,

$$hav\ z = cos\ L\ cos\ D\ hav\ t + hav\ (L \pm D)$$

where the symbol, hav, represents the haversine, or the sine squared of half the angle. In this formula all the quantities are positive, and the work is therefore straightforward and the possibility of confusion and error is reduced to a minimum.

The essential tables are:

Table C, or the Cosine table:—this is a table of logarithmic cosines of all angles from 0° to 90° and is used in connection with the latitude and declination. The quantities are given to five place decimals for single minutes of arc (1′) and can be found by interpolation to any fraction of a minute.

Table H, or the Haversine table:—this is a

table of logarithmic haversines of all angles from 0° to 180°; the quantities being given directly to single minutes of arc (1′), and in some cases to ¼′. As this table is used in connection with the hour angle, the arguments are given in terms of time as well as in terms of arc. The functions, corresponding to any hour angle, from 0° to 180°, can therefore be taken directly from the table without transferring the time into arc. By interpolation the quantities can be found for any hour angle down to seconds of time.

Table N, or Natural Haversines:—this is a table of natural haversines for all angles from 0° to 90°, and is arranged in parallel columns with the H table, so that at one opening of the book both the logarithmic and natural haversines of a given angle may be obtained. By this arrangement also the natural haversine corresponding to a logarithmic haversine can be found without the necessity of a table of logarithms of numbers.

The formula gives the zenith distance instead of the altitude, and this zenith distance is found

from the three known quantities, L, D, and t, by three simple and direct operations as follows:

1. From the C table take out the C's corresponding to to the latitude, L, and to the declination, D; from the H table take out the H corresponding to the hour angle, t. Add these three quantities together and call the sum, S.

   These three quantities are always positive and can be taken directly from the tables to the nearest minute of arc (1′) or four seconds of time (4ˢ) without interpolation. S must always be less than 10, and if the sum appears greater than this figure, then subtract 10 or 20 from the sum as the case may necessitate.

   Formula:

$$S = C_{L} + C_{D} + H_{t}$$

2. Enter the H table with the sum, S, and in the adjacent parallel column take out the corresponding N, which may be designated as N_{S}. In all ordinary cases, working to single minutes, this can be done without interpolation. If greater accuracy is deemed essential, the final figure must be found by interpolation.

3. If the latitude, L, and the declination, D, are of opposite names (one North, the other South), add them together; if they are of the same name (both North, or both South) subtract the smaller from the larger. From the N table take out the N corresponding to this angle, which may be designated as N_{LD}. Add this N_{LD} to the N_S found in the second operation and the sum is the N_Z corresponding to the required zenith distance. With this N enter the N table and take out the zenith distance directly to the nearest minute of arc.

Formula:

$$N_Z = N_{LD} + N_S$$

These three operations require a total of only 47 figures and give the result to the nearest minute of arc without interpolation of any kind. One step in the calculation leads directly to the next, all the quantities are positive, there are no confusing precepts to remember, and every problem is worked out in exactly the same manner.

The following typical example is given in order to illustrate the way in which every sight can be worked.

At sea November 9th in D. R. latitude 34° 20′ N., and longitude 4^h 58^m W., an observation was made at Greenwich mean time, 8^h 15^m 1^s, and the true altitude of the sun found to be 17° 56′. Calculate the altitude of the sun and find its azimuth.

From the "Nautical Almanac" find the declination of the sun and the equation of time. Correct the observed time by the equation of time, and from the result subtract the D. R. longitude, and thus get the ship's apparent time, or the hour angle, t. The three known, or given, quantities with which the calculation starts are, therefore, found to be:

$$L = 34° 20′ \text{ N.}$$
$$D = 16° 56′ \text{ S.}$$
$$t = 3^h 33^m 5^s.$$

The calculation of z and h is as follows:

1. $C_L = 9.91\ 686$ $\quad L \pm D = 51° 16′$
$C_D = 9.98\ 075$
$H_t = 9.30\ 310$
$S = 9.20\ 071$

2. $N_S = 0.15\ 879$
3. $N_{LD} = 0.18\ 715$
$N_Z = 0.34\ 594$

$z = 72^\circ\ 3'$
$h = 17^\circ\ 57'$

and this is the required altitude.

The azimuth of the sun can be found from several different tables; from the "Azimuth Tables" of the Bureau of Navigation, or from special tables contained in the "Sumner Line Tables," or in the "Line of Position Tables." These latter two are similar in form and were specially prepared for use in the St. Hilaire method and are very simple and easy to use. They depend upon the fact that the four quantities, hour angle and altitude, azimuth and declination, are connected together in pairs; so that the altitude holds the same relation to the hour angle as the declination does to the azimuth. A tabulation of this relation between altitude and hour angle is also, therefore, a tabulation of the relation between declination and azimuth.

This relation is tabulated in the Azimuth Tables for all altitudes between 0° and 80°, and for all hour angles from 0 hours to 12 hours. The various tabular zenith distances (or altitudes) are found in the first column at the left hand side of the page and the hour angles are found at the top of the page and are tabulated at a uniform interval of six (6) minutes. The declination is found in the right-hand column and the intervals of tabulation are, of course, the same as those for the zenith distance. The azimuth is at the bottom of the page and is given for a uniform interval of one and a half ($1\frac{1}{2}°$) degrees, corresponding to the six-minute interval of the hour angle.

To find the azimuth from the table, when the three other factors, hour angle, zenith distance, and declination, are given, proceed as follows:

1. Find at the top of the page the column at the head of which appears the hour angle nearest to that given, follow this column down to the horizontal line on which appears the given zenith distance at the left of the page, and at the intersection of this

horizontal line with the hour angle column, find the tabular ratio, r.

2. Find the given declination in the right-hand column of the table, and follow this declination horizontally across the page until a column is found with a ratio equal to or very nearly equal to r. At the foot of the column in which this ratio appears will be found the required azimuth.

The azimuth thus found is always to be counted from the north point in the northern hemisphere, and from the south point in the southern hemisphere. It will be east or west in either case as the hour angle is east or west.

To find the azimuth in the above example, enter the table with the hour angle, $3^h\ 33^m$, and opposite the zenith distance, 72°, find the ratio 843, by simple interpolation. Again enter the table with the declination, 17°, and find the nearest ratio as 846, and the corresponding azimuth at the foot of the column is 126°. Hence the required azimuth is N. 126° W.

The Sumner line of position may now be drawn. On the chart, Fig. 4, plot the D. R. position, O, in latitude 34° 20′ N. and longitude

74° 30′ W., and through this point draw a line running N. 126° W., for the azimuth, or bearing of the sun. As the observed altitude, 17° 56′, is one minute (1′) smaller than the calculated altitude, 17° 57′, the most probable position of the ship will be one mile (1′) further from the subsolar point than the D. R. position. Therefore, prolong the line representing the azimuth of the sun one mile (1′) through the point O to S, and through S draw the Sumner line at right angles to the bearing of the sun. The ship will be somewhere on this line. The prolongation of the Sumner line to the north and west passes a few miles to the north of Cape Hatteras, and therefore this observation gives the bearing of this most important point.

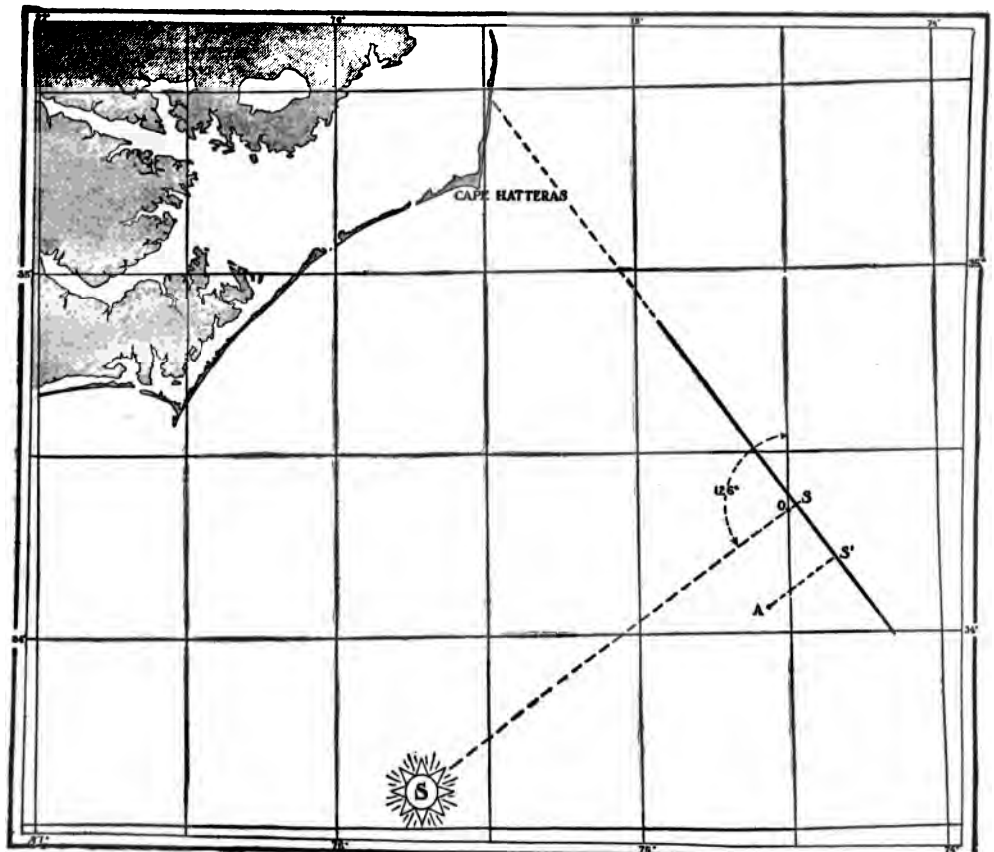


Fig. 4. A Sumner Line and Aquino's Assumed Position

b. *Solutions by Double Entry Tables*

Many attempts have been made to do away with all logarithmic work and to solve every problem that may arise in navigation by means of tables of auxiliary angles. The most successful of these are the tables prepared by de Aquino under the title "Altitude and Azimuth Tables." These are based on methods devised and tables prepared by Sir William Thomson (Lord Kelvin) in 1876. The underlying principle of this method is the division of the fundamental spherical triangle (Fig. 3) into two right-angled spherical triangles, and the tabulation of the solutions of these.

The original and fundamental equation of navigation involves four quantities, three of which, L, D, and t, are known; the fourth, h, is found from these by calculation. Now it is clear that, for certain definite values of L, D, and t, h can be calculated once for all and the values tabulated. Were such a tabulation once made, there would be no need of further calculations, for, by

entering the table with the known values of L, D, and t, the required value of h would be obtained at once by inspection. Unfortunately, however, each of the three known quantities may have any value from 0° to 90°, and in actual practice sights should be worked to single minutes. To tabulate every possible combination, therefore, would require 157,464,000,000 separate entries in the tables. By dividing the fundamental triangle into two right-angled spherical triangles, however, the problem may be reduced, as shown by Lord Kelvin, to the solution of two equations, each involving three quantities instead of four as in the original equation. If precision to single minutes be required, then each equation would give rise to 29,160,000 possible solutions, and tables, arranged for this degree of accuracy without interpolation, would contain some 58,320,000 entries, and would require some 58,320 pages, allowing one thousand entries to a page. Such tables being impossible, tables are constructed by using intervals of one-half degree (½°) for one quantity and single degrees (1°) for the other. In this way the tables are reduced to 32,400 en-

tries, and require only 120 pages; but to use these reduced tables, elaborate interpolations must be made, or a system of approximations used.

The two auxiliary equations, the solutions of which are tabulated, are as follows:

$$\sin a = \cos D \sin t$$
$$\cot b = \cot D \cos t$$

where a and b are auxiliary quantities. From these auxiliaries h can be found from an equation of identically the same form as the first of these, namely:

$$\sin h = \cos a \sin B$$

where B can be found directly from the latitude, L, and the auxiliary, b. The tabulation of the first equation for finding a serves also, therefore, for finding h.

In Aquino's tables a and b are tabulated for various combinations of D and t; D being taken at intervals of thirty minutes of arc (30′), and t at intervals of one degree, or four minutes of time (4^m). For all cases that arise in practice, therefore, interpolation must be made and these interpolations are complicated, for the tables are

double entry tables. This is shown by the following example, which is the same as that of the former section.

The given quantities are:

$$L = 34^\circ\ 20'\ N.$$
$$D = 16^\circ\ 56'\ S.$$
$$t = 53^\circ\ 15'$$

In Aquino's tables under $D = 16^\circ\ 30'$ and $D = 17^\circ\ 0'$ are found the a's corresponding to the nearest values of t, namely 53° and 54°, as follows:

	16° 30′	17° 00′
53°	49° 58′	49° 48′
54°	50° 52′	50° 41′

To find the correct value of a, one must first interpolate in each of these columns and find the values corresponding to 53° 15′. The difference in the first column is 54′ and one quarter of this is 13.5,′ which applied to 49° 58′ gives 50° 11.5′. Interpolating between the two numbers in the second column, one finally obtains for the two values of a,

For D = 16° 30′ a = 50° 11.50′
" " = 17° 00′ " = 50° 1.25′

One must now interpolate between these two values to find that corresponding to 16° 56′, and thus is finally found the true value of a as,

$$a = 50° \ 2.60'$$

A similar set of interpolations from the second table gives the correct value of b as,

$$b = 26° \ 58.4'$$

As D and L are of contrary names,

$$B = 90° - (L + b),$$

and one thus finds:

$$B = 28° \ 41.6'$$

With this value of B and the value of a, one now enters the first table and by a similar system of double interpolation finds for h the value,

$$h = 17° \ 58'$$

which agrees very closely with the value found by the logarithmic calculation of the former section.

To use these tables of Aquino in this direct manner, one must thus make three double interpolations, each one of which involves three separate simple interpolations. Besides the additions and multiplications involved, these interpolations require the writing down of at least 96 figures, thus making this method considerably longer than the direct logarithmic method of the former section.

In order to avoid these long interpolations and to make the use of the tables practicable, Aquino makes use of a device, used by Johnson and others, in which an assumed approximate position is used instead of the position by dead reckoning. This is perfectly legitimate, for the "D. R." position itself is nothing but an assumed position, and the line of position can be found as readily from one as from the other. This assumed position is so taken as to make the auxiliaries a and B even figures for which the tables are computed, and the method by which the assumed position is selected can best be shown in an example. For this purpose take the example just worked and the given quantities are:

$$L = 34^\circ\ 20'\ N.$$
$$D = 16^\circ\ 56'\ S.$$
$$t = 53^\circ\ 15'$$

Enter the tables with D as 17° and t as 53°, these being the nearest figures for which the tables are computed, and take out the approximate values of a and b to the nearest whole degree, as,

$$a = 50^\circ \ : \ b = 26^\circ$$

With these values of a and b as arguments, enter the table and take out the values of D, which most nearly correspond to the true value, namely, 16° 56′. These values are,

For $b = 26^\circ$	$D = 16^\circ\ 22'$
" " $= 27^\circ$	" $= 16^\circ\ 58'$

Now interpolate and find the value of b, which corresponds to the true value of the declination, D. This is a simple interpolation and one easily finds that:

$$\text{For } D = 16^\circ\ 56' \quad b = 26^\circ\ 56.7'$$

The value of t corresponding to these values of

D and b is also found by simple interpolation to be,

$$t = 53^\circ\ 11.8'$$

Now, in order to do away with any further interpolations, this t is assumed to be the hour angle of the sun and the longitude of the place found, where this would be true. This place is the same number of minutes of arc east or west of the D. R. position, as this t is greater or less than the original t. In this case the new assumed place will be 3.2′ west of the D. R. position.

Further, a new latitude is assumed such that B will be a whole number of degrees. As B is found by subtracting the sum of L and b from 90°, one sees that in this special problem the assumed latitude must be 34° 3.3′, in order that B may be equal an even 29°.

Now, with

$$a = 50^\circ \text{ and } B = 29^\circ$$

the corresponding value of h may be taken directly from the table without interpolation, as:

$$h = 18^\circ\ 10'.$$

At the same time that h is thus found, the azimuth may be obtained from the adjacent column of the tables, corresponding to $a = 50°$ and $B = 29°$, as:

$$\text{Azimuth} = 53°\ 44'$$

measured from the south point. This, however, is usually written as N. 126° 16′ W.

The altitude, h, as thus computed will not be the same as that found by the logarithmic method; for the two positions, from which the sun is supposed to be viewed, are different in the two methods. The Sumner lines as plotted by the two methods should agree very closely. This is clearly shown in Fig. 4, in which O is the "D. R." position, for which the altitude is calculated in the logarithmic method, and A is the "assumed" position for which the altitude is found in the Aquino method. LL′ is the Sumner line, and it is readily seen that, if the difference between the distances AS′ and OS is equal to the differences between the two altitudes, as found by the two methods, then the two Sumner lines will exactly coincide and the two methods will give identically the same result. In applying the Aquino

method it must be remembered, however, that S is the most probable position of the vessel and not S′, which is the point from which the Sumner line is plotted. That is, in the Aquino method, after the Sumner line is plotted by laying off the distance AS′ and drawing the line perpendicularly to the direction of the sun, the most probable position of the ship is found by projecting the D. R. position perpendicularly upon the Sumner line so drawn.

Used in this way, the Aquino method is very short and requires few figures. If all the figures, necessary to the interpolations, be written down, it will be found that this method, however, requires more actual figures than the logarithmic method. Moreover, in this method there are more chances of error, the auxiliary *B* is not always determined in the same way, being dependent upon the relation between L and the auxiliary *b*. With the precepts necessary to determine *B* and with the interpolations, this might be termed a fussy method, as contrasted with the simple, straightforward work of the logarithmic method.

c. *Mechanical Solutions*

There have been many attempts to solve all the problems of navigation by means of mechanical devices, or by sets of plotted curves. In general these have not been successful, for in nearly every instance the old time methods were used. The new SAINT HILAIRE method, however, lends itself readily to a mechanical solution. A computing machine has been designed which solves, mechanically, the entire logarithmic calculations necessary in working out a sight by this modern method. With this instrument the calculations are actually performed with a few direct and simple movements, and with an accuracy equivalent to that obtained with four place logarithmic tables. Such accuracy is sufficient for "off shore" navigation on board ship, and is ample for any work on airplanes, or airships.

In principle this instrument is essentially a circular slide-rule. It consists of a circular disc, on which are engraved a series of concentric

graduated circles, a rotating transparent circle, and a movable arm, which arm may be moved separately or clamped to the rotating circle. The graduated circles on the disc are arranged in two divisions; those in one division being used to find the altitude, that in the other to find the azimuth. These circles are arranged as follows:

a. ALTITUDE DIVISION

In this division there are a total of seven graduated circles, but three of these form a sub-group and two others are also of similar character, so that the seven circles may be considered under four heads.

1. *Latitude and Declination Circle.*

LATITUDES, to degrees and fractions, are shown to the left-hand side of the zero line. The graduations in this circle are spaced proportionately to the logarithmic cosines of the corresponding angles, and the graduation is carried to 70°. For low latitudes the scale is such that divisions smaller than single degrees (1°) cannot be shown, but as the scale becomes more and more open with increasing latitudes, the degrees are subdivided.

DECLINATIONS, to degrees and fractions, are shown to the right-hand side of the zero line. As the declination of the sun never exceeds 23½°, this graduation ends at 30°; but, in order to make the device applicable to stellar observations, the declinations (greater than 30°) of the principal navigational stars are shown.[1] Each such special stellar graduation is indicated by a number in a small circle, the number being the number of the star in a special star list. All these declination graduations are spaced proportionately to the logarithmic cosines of the corresponding angles, and for small declinations the scale is divided to single degrees only. For larger declinations the degrees are subdivided.

2. *Hour Angle Circles.*

HOUR ANGLES, between 15 minutes and 12 hours, are shown in three circles. The graduations in these circles are spaced proportionately to the logarithmic haversines of the corresponding hour angles. Up to six hours the divisions on the scale are at intervals of single minutes of time; beyond six hours, however, as the scale becomes more and more contracted, the time intervals are made longer.

[1] In an improved type of machine, the graduations are extended to 70°, making it applicable to all stars, the declinations of which are less than this.

2-a. Shows all hour angles, to single minutes, between $2^h\ 27^m$ and 6 hrs.; and all hour angles between 6 hrs. and 12 hrs. at various longer intervals.

2-b. Shows all hour angles, to single minutes, between $2^h\ 27^m$ and $0^h\ 46^m$.

2-c. Shows all hour angles, to single minutes, between $0^h\ 46^m$ and $0^h\ 14^m$.

3. *Logarithm and Number Circles.*

LOGARITHMS of numbers are shown on Circle 3-a. This circle is numbered from 50 to 500 and the divisions are spaced logarithmically. The division, or graduation, numbered 500 is used also for 50 and for 5, and similarly each other graduation may represent the number as shown and that number divided by 10 and also by 100.

NUMBERS are shown on Circle 3-b. The divisions are equally spaced and are numbered consecutively from 0 to 500.

4. *Altitude and Zenith Distance Circle.*

ALTITUDES, to degrees and fractions, are shown on the outer side of this circle. The graduations are spaced proportionately to the natural haversines of the corresponding angles. For very great altitudes the scale is such that divisions smaller than single degrees

(1°) can not be shown, but the scale opens very quickly as the altitudes decrease and the degrees are subdivided, so that all altitudes less than 60° are shown to ten minutes (10′) of arc.

ZENITH DISTANCES, to degrees and fractions, are shown on the inner side of the Altitude Circle.

b. AZIMUTH DIVISION

In this division there is only one graduated circle, but this circle is variously lettered for hour angles, altitudes, and azimuths. The graduations are spaced proportionately to the logarithmic sines of the corresponding angles, and are given to degrees only from 6° to 90°. Altitudes or azimuths smaller than 6° are not given.

HOUR ANGLES are shown in two series:

From 0^h24^m to 6 hrs., at four minute intervals, proceeding towards the right.

From 6 hrs. to 11^h36^m, at four minute intervals, proceeding towards the left.

Hour angles less than 24 minutes are not shown.

ALTITUDES, to single degrees, are shown just

outside the graduations and inside the hour angles.

AZIMUTHS, to single degrees, are shown in two series on the inner side of the graduations:

From 6° to 90° proceeding towards the right.

From 90° to 174° proceeding towards the left.

METHOD OF USING INSTRUMENT

With this instrument the calculation of the altitude, as required by the Saint Hilaire method, is reduced to four (4) simple and direct movements, while the azimuth may be found by two (2) movements. The entire calculation is made on the instrument, the work is straight-forward and requires the writing down of one figure only.

ALTITUDE COMPUTATION

1. Set the line of the movable circle, C, on the declination, D, of the observed body as found on the Declination Circle, (1) right.

Set the line of the movable arm, M, on the latitude, L. Clamp the arm and circle together.

Example:

At sea, at the time an observation was made,

the declination of the sun was given by the Almanac as 16° 56′ S. Set the line of the movable circle at the nearest division of the declination circle, 17°.

The D. R. latitude of the ship was 34° 20′ N. Set the line of the movable arm to 34° 20′ on the latitude circle, and clamp the arm and circle together.

On the completion of this first movement, the positions of the circle and arm will be as shown on the diagram. (Fig. 5.)

2. Move the circle and arm thus clamped together until the line of the circle, C, coincides with the hour angle, t, of the body observed. Read the number under the line of the arm, M, on the Logarithmic Circle, 3-a.

If the hour angle is found on Circle 2-a, this number is to be taken as read;
If the hour angle is found on Circle 2-b, this number is to be divided by 10;
If the hour angle is found on Circle 2-c, this number is to be divided by 100.

Exceptions: If the declination and latitude be such that the angle between the circle, C, and the arm, M, when clamped in the first position, is *less* than the latitude angle of 60°, then, when the hour angle is such that the line of the circle, C, is to the left of the zero line, and the line of the arm, M, is to the right of latitude 60°, (that is, the circle and arm are both included in the latitude angle of 60°), if the hour angle is found on Circle 2-b, the number is to be taken as read; and if the hour angle is on Circle 2-c, the number is to be divided by 10 only.

If the declination and latitude be such that the angle between the circle, C, and the arm, M, when clamped in the first position *exceeds* the latitude angle of 60°, then, when the hour angle is such that the line of circle, C, is to the right of the zero line and the line of the arm, N, is to the left of latitude 60°, (that is, the latitude angle of 60° is entirely included between the lines on the circle and the arm) the resultant number, as above found, is to be divided by 10.

With the sun, or with any star the declination of which is less than 23°, this latter case can only happen in latitudes higher than 57°. With stars of greater declination than 60°, this case might occur in any latitude; while for any star between 23° and 60° declination, there will be a limiting latitude below which this case cannot occur.

Example:

From the chronometer time of the observation

and "D. R." longitude, the hour angle of the sun was found to be $3^h\ 33^m$. Move the circle and arm clamped together until the line of the circle, C, is over this hour angle on the graduated circle, 2-a; and under the line of the arm, M, find on circle 3-a the number 159.

On the completion of this second movement, the positions of the circle and arm will be as shown in the following diagram: (Fig. 6.)

8. Set the line of the circle, C, to zero, and set the line of the arm, M, on the zenith distance circle (4) right, to the angle corresponding to the difference between the latitude and the declination, if both are of the same name (both north or both south); if they are of opposite names (one north and one south) then use the sum of the two and set the arm on the angle corresponding to this sum.

Clamp the circle and arm together.

Example:

As the declination and latitude are of opposite names the sum is taken as 51° 16′. The circle is first set to zero and then the arm is moved until

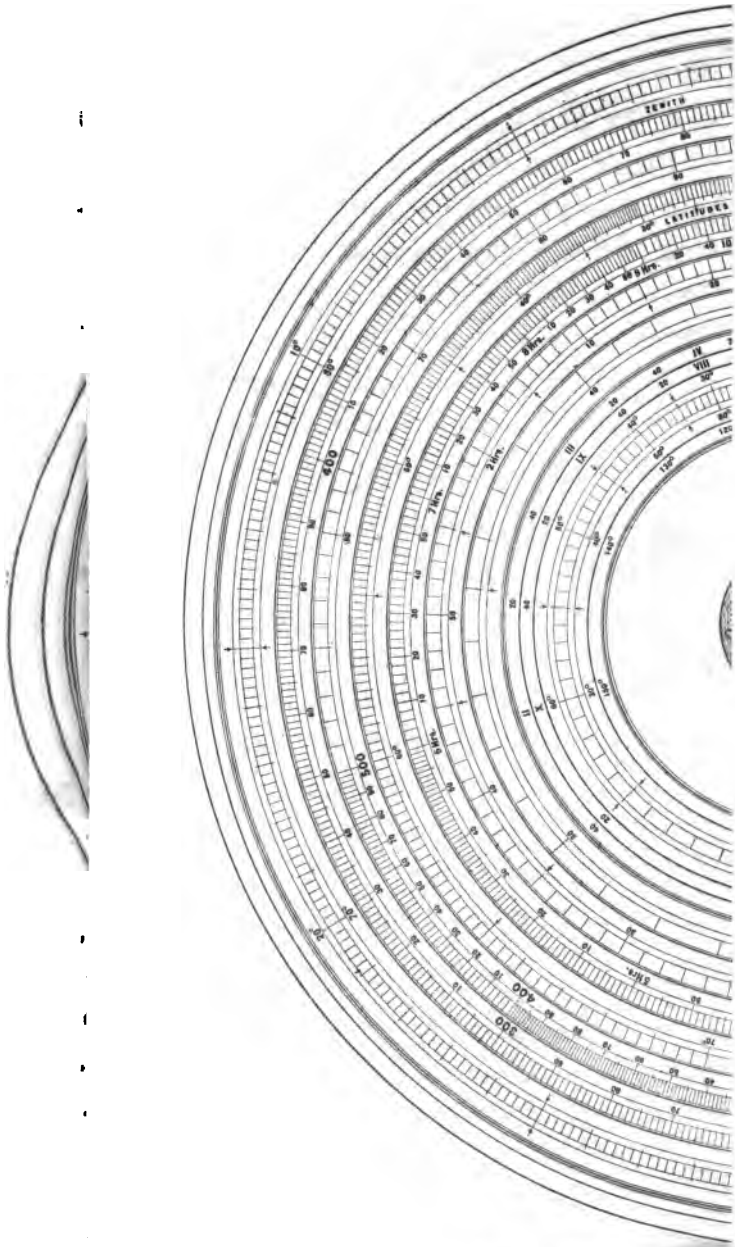
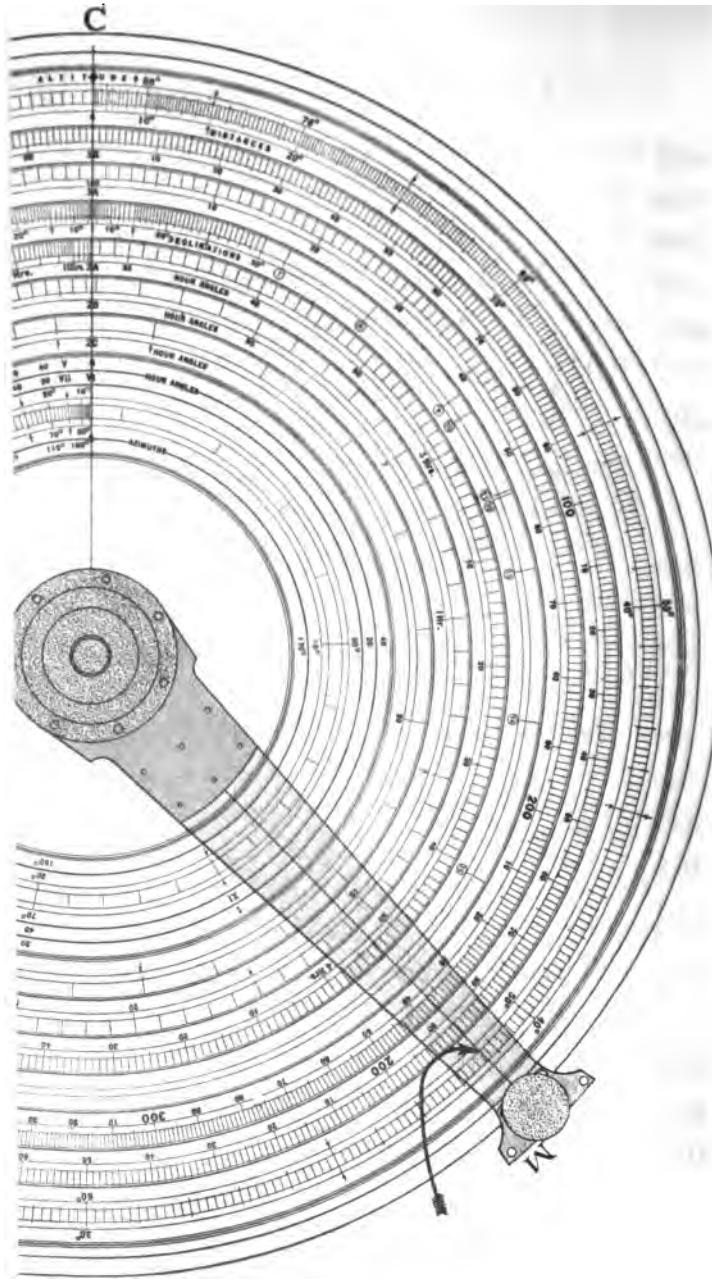


Fig. 7. Position



ing for Difference Between Latitude and Declination

the line is over 51° 16′ on the zenith distance circle (4) right, and the arm and circle are clamped together.

On the completion of this third movement, the position of the circle and arm will be as in the annexed diagram. (Fig. 7.)

4. Move the circle and arm thus clamped until the line of the circle, C, is over the same number on the number circle, 3-b, as was found on the logarithm circle, 3-a, by operation 2: or the number reduced in the special cases.

The required altitude will now be found un- the line of the arm, M, on the altitude circle, (4).

Example:

Move the clamped circle and arm from position 3, until the line of the circle, C, is over 159 on the number circle, 3-b: under the line of the arm, M, will be found the required altitude, 17° 55′.

On the completion of this final movement, the position of the arm and circle will be as in the diagram below. (Fig. 8.)

AZIMUTH COMPUTATION

1. Set the line of the circle, C, on the hour angle, t, of the body on the Azimuth Circle, (5). Set the line of the arm, M, on the altitude, H, of the body on the Azimuth Circle, (5). Clamp the arm and circle together.

Example:

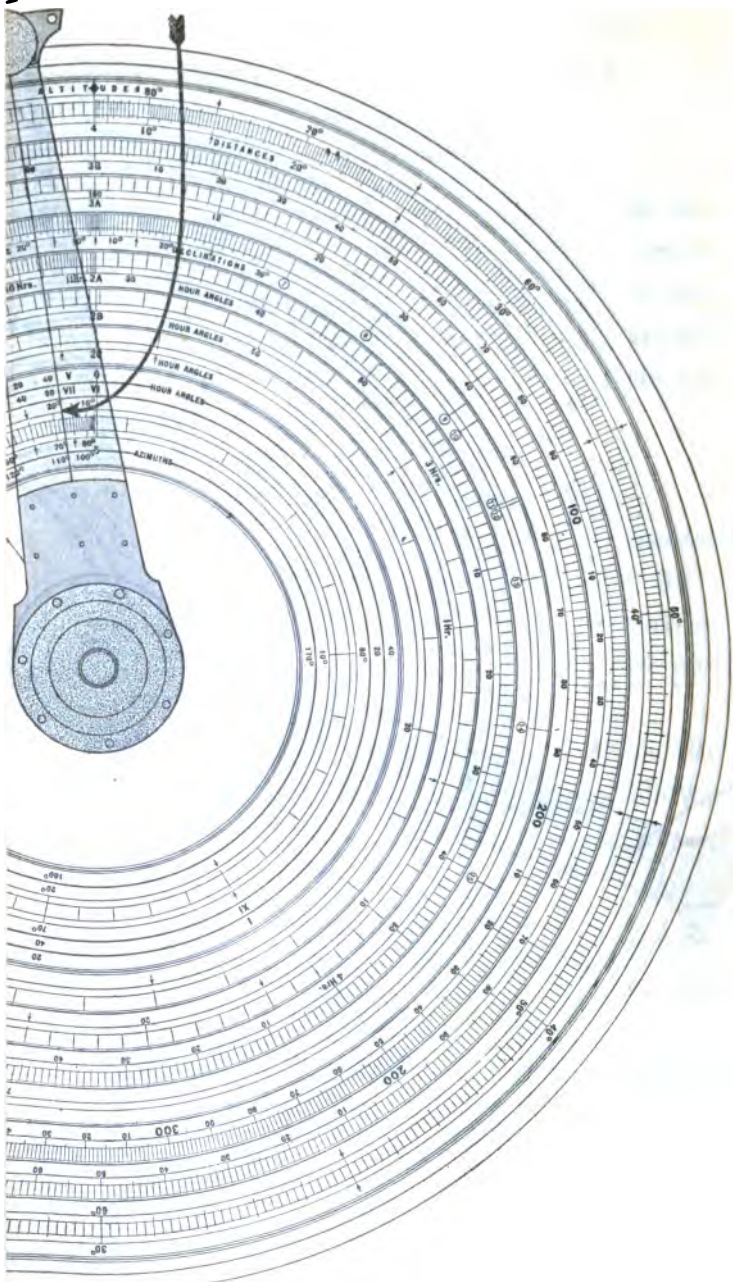
Set the line of the circle, C, on the nearest graduation to the hour angle, t, namely $3^h\ 32^m$: set the line of the arm on 18°, the graduation nearest to the altitude; clamp the arm and circle together.

On the completion of this first movement, the positions of the circle and arm will be as shown on the diagram. (Fig. 9.)

2. Move the circle and arm thus clamped together until the line of the arm, M, is over the declination, D, of the body on the Azimuth Circle (5).

   Under the line of the circle read the required azimuth.

Fig. 9. **Position 1.-**



—Setting for Hour Angle and Altitude

The required azimuth may be either greater or less than 90°, and, in general, the conditions under which the observation was made will determine this at once. In some cases, when the observed body is nearly east or west, some confusion might occur, and then the following rules will act as guides.—

1. When declination and latitude are of contrary names (one north, the other south), then the azimuth will always be greater than 90°.

2. If the declination be greater than the latitude, both of the same name, then the azimuth will always be less than 90°.

Example:

Move the clamped arm and circle until the line of the arm is over 17°, the graduation nearest to the declination; under the line of the circle is found the azimuth as 54°, or 126°.

As the latitude and declination are of contrary names, the azimuth is greater than 90° and hence it is N 126° W.

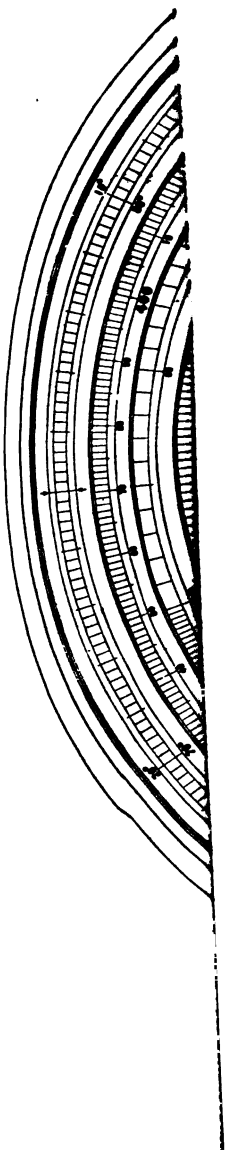
On the completion of this second movement, the final positions of the arm and circle are as shown in the annexed diagram. (Fig. 10.)

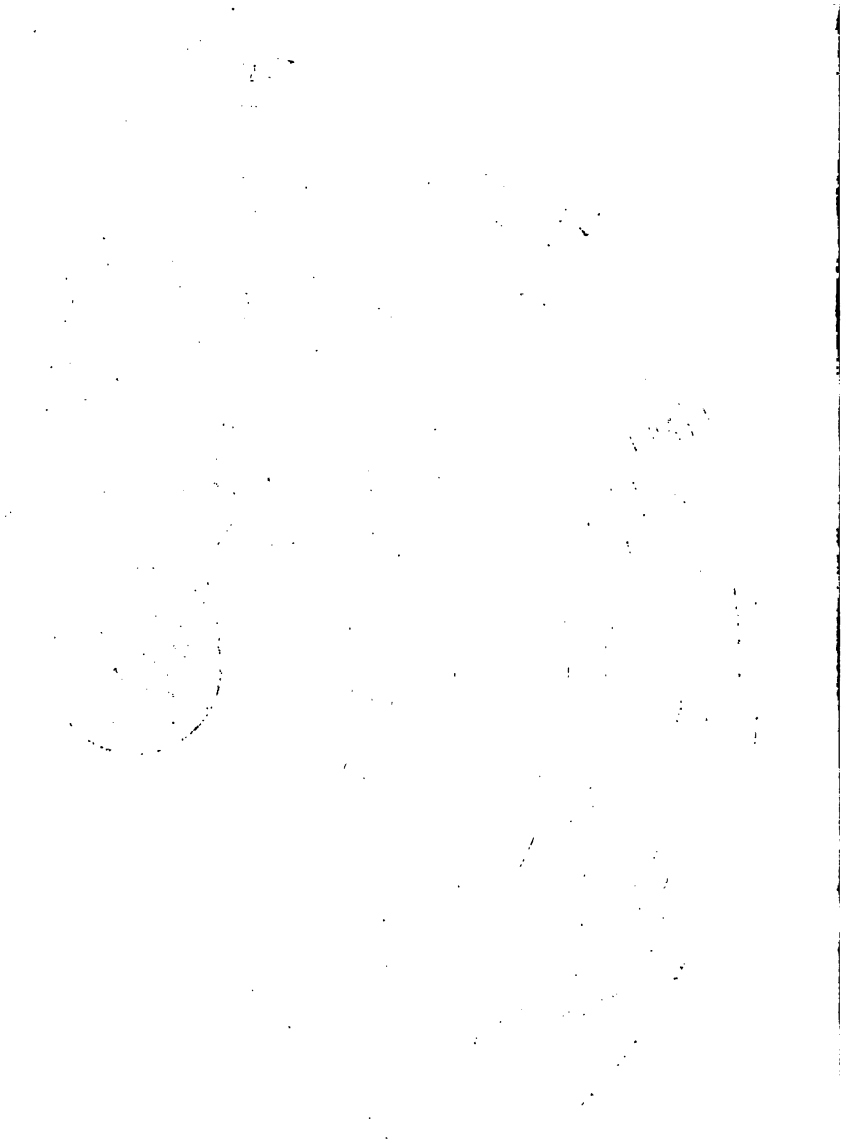
Comparison of the Three Methods.

Either of the above three methods may be used to work out any, or every, sight taken by a navigator at sea or in the air. The logarithmic method, however, is the most accurate and should always be used when navigating dangerous waters and when extreme and minute accuracy is necessary. But for ordinary, everyday "off shore" navigation either one of the three methods is sufficiently accurate and either one may be used with equally good results.

In air-craft, however, the mechanical method is the only one available, for it would be extremely difficult, if not impossible, for an aviator to use the books and tables required by either of the other two methods.

In order to compare the three methods, the following standard example is worked out in each way; all the work and figures being shown in each case.





At sea, January 15, 1915, in latitude 30° 10′ N. and longitude 45° 15′ W. by dead reckoning, at chronometer time $11^h\ 32^m\ 50^s$, the altitude of the sun's lower limb, bearing south and east, was 17° 40′ 50″. Index correction 1′: height of eye 29 feet: chronometer slow $7^m\ 10^s$ on Greenwich mean time.

The following work is common to all methods:

Chronometer time	11^h	32^m	50^s
Correction		7	10
Greenwich M. T.	11	40	0
Longitude, W.	3	1	0
Local M. T.	8	39	0
Equation of time	−	9	16
Apparent time	8^h	29^m	44^s
Hour angle	3^h	30^m	16^s

Declination	
At noon	21° 16′ 22″ S.
Corr.	+ 9″
Decl.	21° 16′ 31″ S.

Equation of Time	
At noon	$9^m\ 16.9^s$
Corr.	− .3^s
	$9^m\ 16.6^s$

Whence the data:

L	= 30° 10′ N.
D	= 21° 16.5′ S.
L ± D	= 51° 26.5′
t	= $3^h\ 30^m\ 16^s$

Observed Altitude	
Observed	17° 40′ 50″
Index	+ 1′ 0″
Refract. etc.	+ 8′ 7″
h =	17° 49′ 57″

The following work is peculiar to each method:

1. *Logarithmic Method.*

$$\begin{aligned} C_D &= 9.93\ 680 \\ C_L &= 9.96\ 934 \\ H_t &= 9.29\ 244 \\ \hline S &= 9.19\ 858 \end{aligned}$$

$$\begin{aligned} N_S &= 0.15\ 797 \\ N_{LD} &= 0.18\ 835 \\ \hline N_Z &= 0.34\ 632 \\ z &= 72°\ \ 6' \\ h &= 17°\ 54' \end{aligned}$$

$$\begin{aligned} \text{Ratio} &= 835 \\ \text{Azimuth} &= \text{N. } 129°\ \text{E.} \end{aligned}$$

2. *Aquino Method.*

$$\begin{aligned} t &= 52°\ 34' \\ a &= 48° \\ b &= 32° \\ \hline D &= 20°\ 46' \end{aligned}$$

$$\begin{aligned} b &= 32° \\ \text{correction} &\qquad 50.9' = 1.67 \times 30.5 \\ \hline b &= 32°\ 50.9' \end{aligned}$$

t =	52° 38′
correction	17.8′ = 0.35 × 50.9
t =	52° 55.8′
Greenwich A. T.	7° 19′
Assumed Long.	45° 36.8′
b =	32° 50.9′
C =	63°
Assumed Lat.	30° 9.1′
h =	17° 41′
Azimuth =	51° 16′
=	N. 128° 44′ E.

3. *Mechanical Method.*

number =	157.6
h =	17° 55′
Azimuth =	N. 129° E.

A comparison of these three methods shows the following number of book openings and auxiliary figures necessary in each to obtain the required altitude and azimuth:

Logarithmic	8 openings and 45 figures.
Aquino	2 openings and 64 figures.
Mechanical	1 opening and 4 figures.

The accompanying figure shows the Sumner line, as plotted on the chart, by each of these three methods. The lines, as determined by the logarithmic and Aquino methods, differ by less than one-eighth (1/8) mile; while that determined by the mechanical method differs from that of the logarithmic method by about one (1) mile. In this figure, O represents the D. R. position, O′ the Aquino assumed position, and LL′, AA′,

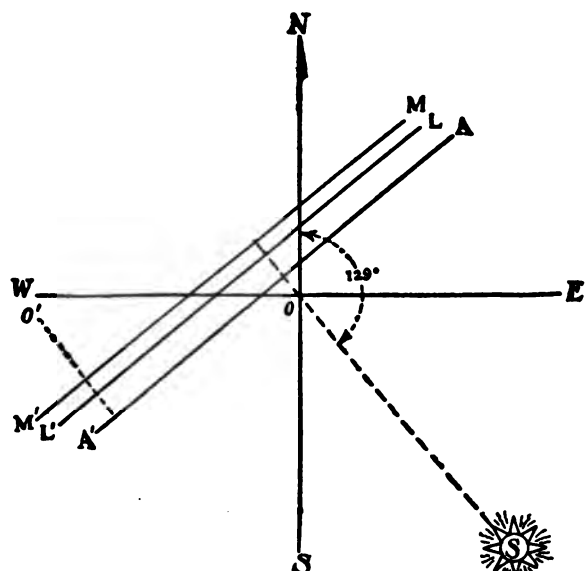


Fig. 11. Comparison Between the Sumner Lines as Found by the Logarithmic, the Aquino, and the Mechanical Methods

and MM′ the Sumner lines as determined respectively by the logarithmic, the Aquino, and the mechanical methods.

Additional Example—Mechanical Method.

At sea, June 27, in D. R. latitude 49° 40′ N. and longitude 5° 19′ W., the true altitude of the centre of the sun was 58° 9′ at Greenwich mean time $10^h\ 51^m\ 40^s$.

From the "Almanac" was found:

Declination	23° 20′ N.
Equation	$2^m\ 49^s$

And the entire computation follows:

Greenwich time	10^h	51^m	40^s
Longitude		21	16
	10	30	24
Equation		2	49
Apparent time	10^h	27^m	35^s
Hour angle	1^h	32^m	25^s
L ± D	26°	20′	

As the hour angle is found on circle 2-b, the number on the computer is divided by 10:

Number	=	23.9
Altitude	=	58° 0′
Azimuth	=	N. 137° E.

The difference between the observed and computed altitudes is 9′, and the most probable position of the ship, therefore, is 9′ N. 137° E. of the D. R. position. The Sumner line passes through the most probable position and runs N. 47° E., or at right angles to the bearing of the sun.

STELLAR OBSERVATIONS

Observations of stars or planets may be reduced just as simply as those of the sun, provided a sidereal chronometer be used to note the time. Such a chronometer is similar in every respect to an ordinary chronometer, but is regulated to keep "star" or "sidereal" time. This differs from the ordinary mean solar time by 3^m 56.56^s per day; that is, in each 24 hours of solar time there are 24^h 3^m 56.56^s of sidereal time. Twenty-four hours of sidereal time, or a sidereal day, meas-

ures one complete rotation of the earth on its axis; measures the interval of time between two successive passages of the same star across one's meridian. A chronometer regulated to such time, therefore, will always indicate the same hour, minute, and second, when a given star is on the meridian, and the time thus indicated is equal to the star's right ascension.

With a sidereal chronometer regulated to Greenwich time, therefore, one proceeds as follows. Correct the observed time for chronometer error, if any, thus finding the Greenwich sidereal time at which the observation was made. From this subtract the "D. R." longitude to find the ship's sidereal time; from which in turn subtract the right ascension of the star or planet, and the result is the hour angle, t, used in the computation.

From this point on the work of reducing a star sight is identically the same as that of reducing a solar observation.

If the observer be not provided with a sidereal chronometer and is obliged to note the time of observation on an ordinary mean time chronom-

eter, then the process is a trifle more complicated. For this mean time must be converted into sidereal time, and this can only be done by the aid of tables published in the "Nautical Almanac."

The following example illustrates the method of reducing a star sight:

On September 28, in D. R. latitude 46° 20′ N. and longitude 29° 57′ W., at Greenwich sidereal time $2^h\ 8^m\ 41^s$, the observed altitude of α Ursae Majoris, "Dubhe" was 19° 26.8′. Height of eye 20 feet.

From Table IV the position of the star is found:

*'s Right Ascension	$10^h\ 58^m\ 4^s$
Declination	62° 15′ N.

The computation follows:

Sidereal time	=	$2^h\ 8^m\ 41^s$	
Longitude	=	1 59 48	
Local Sid. time	=	0 8 53	
*'s R. A.	=	10 58 4	
Hour angle, t	=	$10^h\ 49^m\ 11^s$	E.
L ± D	=	15° 55′	

On the mechanical computer the declination of the star is shown at 15, and the computation is as follows:

Number	=	314
Altitude	=	19° 30′
Azimuth	=	N. 9° E.
Observed altitude	=	19° 26.8′
Correction	=	−7.1′
True altitude	=	19° 19.7′
Altitude difference	=	10.3′

Through the "D. R." position draw a line running N. 9° E.; on this lay off a distance equal to 10.3′ towards the south. Through the point thus found draw the Sumner line running N. 99° E., and the ship will be somewhere on this line.

CHAPTER III

CORRECTIONS TO BE APPLIED TO THE MEASURED ALTITUDE

THE altitude of the sun, as measured with the sextant, is not the true altitude of that body, and cannot be used directly in calculating the position of the ship. There are certain corrections to be applied to the angle as read from the sextant; and these corrections are due to the limitations under which all observations are necessarily made. The earth is a sphere and is surrounded by a dense atmosphere, and the observer is necessarily situated on, or very near, the earth's surface and must, therefore, view the sun through this atmospheric veil. The rays of light, in passing through this atmosphere, are bent from their direct course, and the sun is never seen in its true direction. Again, the visible horizon from which the altitude is measured, is not the true horizon. The visible horizon changes with the height of

the observer above the earth's surface, but the true horizon of a place is always the same. There are, in all, four of these corrections, refraction, dip of the horizon, parallax, and semi-diameter; and these will be discussed separately.

REFRACTION

Whenever a ray of light passes from a rarer to a denser medium, its direction is suddenly changed. This is very noticeable when light passes from air into water, or from air into glass; and this principle is utilized in the manufacture of prisms and lenses, in the making of the eye-glass and in the construction of giant telescopes. The atmosphere, from its upper limits downwards, is a medium of gradually increasing density; and a ray of light, therefore, coming from the sun to the observer's eye is continually passing from a rarer to a denser medium, and it is, therefore, being continuously bent. Such a ray describes a curved path through the atmosphere and the direction in which one sees the sun is that from which the light finally reaches the eye. The difference between the true direction

of the sun and the direction in which it is thus seen is called the "refraction."

This refraction raises all bodies, it makes them appear higher in the heavens than they really are. It decreases their zenith distances without altering their azimuths. The amount of this apparent lifting changes with the distance of the body above the horizon. At the zenith, or directly overhead, the refraction is practically zero, at the horizon it amounts to nearly two thirds (2/3°) of a degree. The apparent diameter of the sun is only about one half a degree; so that, at the horizon, the refraction is more than the diameter of this body. When, therefore, the lower edge of the sun appears just touching the horizon, the entire body is in reality below the horizon and entirely out of the direct line of vision. One actually sees the sun around a "corner" of the earth.

If the air were always perfectly still and never changed its density, the refraction would always be the same for the same altitudes. But the air is never still, it is continually disturbed by currents and by storms; its temperature, as given by

a thermometer, its pressure, as measured by a barometer, are constantly fluctuating. With every such change the amount of the refraction changes. As the air grows warmer, its refrac-

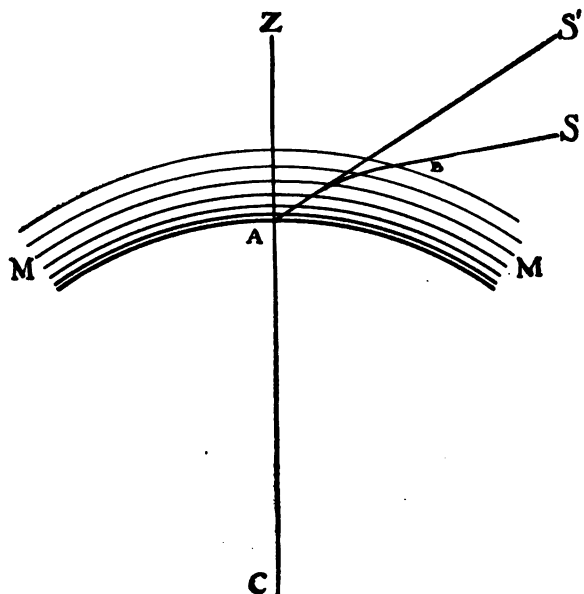


Fig. 12. Refraction

tive power decreases; as it grows denser, the refraction increases. For the accurate work of the astronomer, refraction is extremely troublesome; for the more or less rough work of the navigator

tables have been prepared, as the result of many experiments, which give the amount of refraction with sufficient accuracy.

The refraction is illustrated in the accompanying diagram, in which MM represent the strata of the earth's atmosphere and A the position of the observer on the surface of the earth. A ray of light from the sun, S, enters the atmosphere at B and is bent into the curve BA, reaching the eye of the observer in the direction of the tangent, S′A, of the curve at the point A. To the observer, therefore, the sun appears in the direction AS′, while its true direction is BS. The difference between these two directions is the refraction.

DIP OF THE HORIZON

At sea the altitude of the sun is always measured from the visible horizon, and this visible horizon differs from the true or celestial horizon. This latter is the great circle traced on the heavens by a plane tangent to the surface of the earth at the place of observation; it is everywhere 90° from the zenith. On the other hand, the visi-

ble horizon varies with the height of the observer above the surface of the water; it is always more than 90° from the zenith. As the observer rises

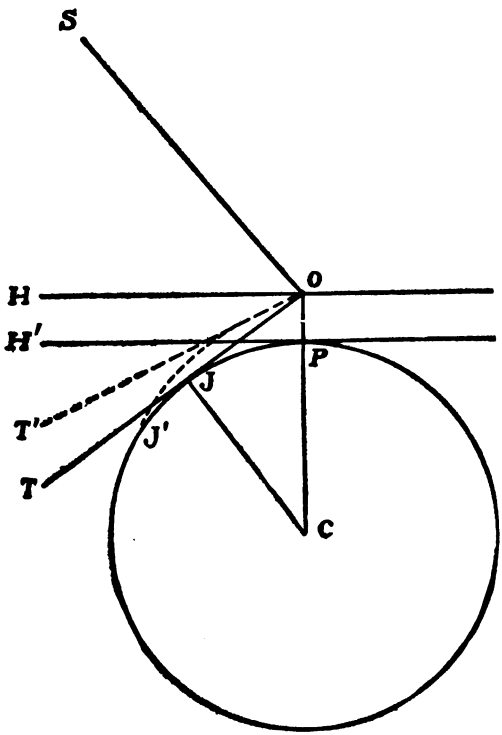


Fig. 13. Dip of the Horizon

higher and higher above the surface, the visible horizon is depressed more and more. From the masthead of a ship at sea, or from an airplane,

one can see considerably more than one half of the heavens. This angular depression of the visible horizon is called the "dip," and its essential elements are shown in the adjacent figure.

In this diagram C represents the centre of the earth, and O the eye of the observer at an elevation PO above the earth surface. PH′ is tangent to the surface at P, and represents the true horizon; the line OH, parallel to PH′, also represents the true horizon, for these two lines PH′ and OH meet the celestial sphere in the same great circle. OT is the visible horizon, for the observer can see any object on the surface between P and J and can observe any heavenly body which is above the line OT. The dip is the angle HOT. And the altitude of the sun, measured from the visible horizon, is always greater than when measured from the true horizon.

The dip depends upon the size of the earth and upon the height of the observer above the sea level. The problem of finding the dip is a simple one in geometry and a very simple and accurate geometrical formula may be obtained quite readily. But such a simple formula would not

be entirely satisfactory in practice, for atmospheric refraction bends the rays of light which come to the observer and elevates the visible horizon. This is shown in the diagram. The rays which start from J′ travel a curved path and reach the observer's eye at O, and the point J′, which is now on the visible horizon, is seen in the direction OT′. Thus the actual dip is not so great as the simple geometrical formula would lead one to believe it should be, and, further, it is evident that the actual sea horizon is farther from the observer than it would be if the earth had no atmosphere.

The amount that the refraction raises the sea horizon is rather uncertain; it depends upon the state of the atmosphere. If along the path of the ray between J′ and O the air is of uniform density, then this path will be a straight line and refraction will have no effect upon the dip. This, however, is very unusual, for the density of air depends upon its temperature, and the different strata of air are seldom at the same temperature. If it is assumed that the temperature of the lowest stratum of air is the same as that of the water,

then when the water is cooler than the air at the observer's station, the air will be denser at J′ than at O, and the ray of light will be bent as shown in the diagram. In this case refraction diminishes the dip. On the other hand, if the water is warmer than the air, then the atmosphere at J′ will be less dense than at O and the ray will be bent downward. In this rather unusual case the refraction will increase the dip, and diminish the distance at which can be seen objects on the surface of the water.

The actual effect of refraction upon the dip can thus be determined by noting the temperature of the water and that of the air at the height of the eye. In fact, tables can be computed which will show the amounts by which the dip must be corrected for various combinations of different temperatures of the air and water. Such tables are of value in unusual conditions, but for the ordinary purposes of navigation an average state of the atmosphere may be presupposed and the effect of refraction upon the dip computed upon this assumption.

All that has been said in regard to navigation

at sea, applies equally to travel in the air. The observer in an airplane measures the altitude of the sun from the visible horizon, and on account of the great altitude of the plane above the surface of the earth, the altitude thus measured is very much greater than the true altitude. For such an observer the dip of the horizon is very great. And the exact amount of this dip will always be more or less uncertain, for it is extremely difficult to locate the visible horizon. At the height of a thousand (1000) feet, the visible horizon is forty-two (42) miles distant; at ten thousand feet (10,000), it is one hundred and thirty miles (130) distant, and at these distances the actual horizon will be obscured by haze, mists, and clouds. The measurement of the altitude is therefore difficult and large errors are apt to occur. For this reason, dip tables prepared for the average state of the atmosphere are sufficiently accurate for all purposes of aerial navigation.

An approximate value of the dip can always be found by taking the square root of the number of feet of the observer's eye above sea-level,

and calling the result the dip in minutes of arc. For an observer nine (9) feet above the surface the dip will be three (3) minutes; for an observer one hundred (100) feet up, it will be ten (10) minutes; for an observer in an airplane at an altitude of one thousand feet (1000), the dip will be thirty-two (32) minutes; at an altitude of ten thousand (10,000) feet, it will be one hundred (1° 40′) minutes.

At an average height of ten feet above the water, a change of seven (7) feet will alter the dip by one minute, while at a height of a thousand feet it requires a change of sixty-five (65) feet to increase the dip by the same amount, and at ten thousand feet altitude the dip will change still more slowly, or at the rate of one minute for each two hundred (200) feet. Hence when the observations are made from the low deck of a vessel, the height of the eye above the water must be accurately known. The rise and fall of the vessel on the seas will appreciably affect the dip. On the other hand, when observations are made from an airplane at high altitudes, the height need not be known with great accuracy, for varia-

tions in height of several hundred feet will make but little difference in the dip.

PARALLAX AND SEMI-DIAMETER

In predicting the position of the sun for noon of each and every day of the year, the "Nautical Almanac" can do so only for an observer in some standard locality. Now by common consent this standard locality, to which the positions of all bodies are referred, is one which no observer has ever yet reached—the centre of the earth. In the technical sense used in astronomy the parallax of the sun is the difference in direction in which it would be seen by an actual observer on the surface of the earth and by an imaginary observer at the centre. If the sun were directly overhead, in the zenith, then the parallax would be zero, for the sun, the real, and the imaginary observers would all be in a straight line, and the two observers would see the sun, therefore, in the same direction. If, however, the sun appear in any other part of the heavens than the zenith it will appear to the real observer lower in the heavens

than to the imaginary observer at the centre, or lower than as predicted by the almanac. The difference between the geocentric, or almanac, place and the actual observed place is the parallax. This parallax must always be added, therefore, to the observed altitude in order to obtain the true geocentric, or almanac, altitude.

The exact amount of this parallax depends upon two things: the distance of the body from the earth, and the altitude of the body above the horizon. It changes, therefore, from hour to hour as the body rises in the heavens, being at its greatest when the body is just rising above the horizon, and at its smallest when the body is on the meridian. In the case of the fixed stars the parallax is absolutely insensible; in the case of the sun it is never more than nine seconds (9″), a quantity almost inappreciable in practical navigation. In the case of the planet, Mars, the parallax may be considerably greater, but never large enough to seriously affect an observation. From the minuteness of this quantity, many distinguished writers on navigational subjects de-

duce the practical result that parallax may be "left out in the cold" without detriment to navigation.

It is different, however, with the semi-diameter of the sun. This is a quantity that must be taken into account in every observation of that body. In preparing tables of the motions of the various bodies of the solar system, the motions of the centres of the bodies alone are considered, and, therefore, the tabulated places of the sun, as given in the almanac, are the positions of the centre of the sun. The apparent diameter of the sun's disc is approximately thirty-two (32′) minutes, and this varies from day to day with the varying distance of the sun from the earth. As it is almost impossible, in making an observation, to estimate the position of the centre of a disc of this size, it is customary to measure the altitude of either the upper or the lower limb of the sun. It is clear, therefore, that the measured altitude of a limb must be corrected by the semi-diameter in order to obtain the true altitude of the centre. This semi-diameter is tabulated in the almanac and its value given for each and every day of the

year. For accurate work the exact value of this quantity should be taken from the almanac for the day on which the observation be made.

The mean value of this semi-diameter is sixteen (16′) minutes and the actual value never differs from this by more than one quarter minute (16.2″). For all the practical purposes of navigation, therefore, this semi-diameter may be assumed as constant and equal to its mean value.

For the average state of the atmosphere the refraction is always the same for the same measured altitude; and as the semi-diameter of the sun may be assumed as constant, the sum of these two corrections is always the same for the same measured altitude. This correction, therefore, is a correction which can be applied, once for all, to the graduations on the arc of the sextant, so that the sextant reading will be the corrected altitude of the centre of the sun. With a sextant so graduated the navigator can dispense with the tabular corrections and forget all about refraction and semi-diameter.

While it is theoretically possible thus to graduate a sextant, a mechanical difficulty is encoun-

tered. The arc of a sextant is usually graduated to ten minutes (10′) and closer readings are made by means of a vernier. This vernier will subdivide the divisions on the arc accurately, only when the divisions are uniform and equidistant, as is the case in all ordinary forms of measuring instruments. With the arc of the sextant graduated so as to include the corrections for refraction, the divisions on the arc are no longer equidistant, being considerably farther apart for small angles than for large ones. For angles smaller than ten (10°) degrees, the change in the spacing of the divisions is so great that the use of a vernier becomes impossible. For all angles, however, greater than fifteen (15°) degrees, the relative spacing is so nearly uniform that a vernier can be used and will give results accurately to within one (1′) minute of arc.

It is, therefore, possible to graduate a sextant between 15° and 90° so that its readings will give the corrected altitude of the centre of the sun, with an accuracy sufficient for all practical purposes. As few observations are made at altitudes less than 15°, these limits suffice in most

cases of actual practice. Such a graduation might be added to the ordinary graduation on the limb of the sextant, and an instrument would then be obtained which could be used, within the above limits, to give the corrected altitudes of the centre of the sun on the special arc, and also capable of measuring any angle in the usual manner on the ordinary arc.

CHAPTER IV

SIMULTANEOUS ALTITUDES

THE observed altitude of any body gives the line of position on which the observer is located. It cannot give his position on that line. But, if the altitude of a second body be taken, this second observation will determine a second line of position on which the observer must also be situated. Being thus on both lines at once, the observer must be at their point of intersection, and his absolute position is fully determined. The two heavenly bodies are used in a manner precisely similar to that in which two light-houses, or any two terrestrial objects, are used to obtain a ship's position by cross-bearings, and the method of "simultaneous" or "double" altitudes might aptly be called the method of "astronomical cross-bearings."

The only time that this simple method of cross-

bearings can be used is at night, for in the daytime but one body, the sun, is visible. But at night many stars are visible, and, at almost any hour, two stars may be selected which will give admirable results. In selecting the stars, it is only necessary to pick out a pair whose azimuths

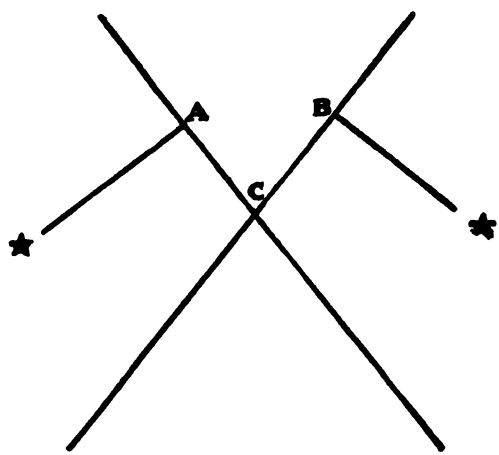


Fig. 14. Simultaneous Altitudes

differ by more than 30°. For the lines of position will intersect at an angle equal to the difference of azimuths of the two bodies, and unless these lines "cut" at an angle greater than 30°, it will be difficult to determine the exact position

of the point in which they cut. The nearer the "cut" is to 90° the more accurate the "fix."

This method of simultaneous altitudes is illustrated in Fig. 14. An observer in approximate latitude 39° observed two stars, one east, the other west, of his meridian. The first observation gave the Sumner line AC, the second the line BC. As the observer must be on both lines, his true position was at C, their point of intersection. His assumed latitude, 39°, was in error by nearly 7′, and the first observation itself would have located the ship at a point nearly 10′ from the true position.

The "Nautical Almanac" gives the positions of some 150 stars; so that at almost any hour of any night a pair of these can be selected, which will serve for such "astronomical cross-bearing."

In the day-time, with only one body, the sun, visible, a modification of this method must be used. If the observer, or the ship, be stationary, the modification is extremely simple. As the sun rises and sets it continually changes its azimuth, and, as the line of position is always at right angles to the direction of the sun, the line

of position will rotate about the observer. If then an observation be made and the Sumner line drawn, and after an interval of an hour and a half or two hours a second observation be made, and a second Sumner line drawn, then these two lines will intersect at a considerable angle and an accurate "fix" will result. If, therefore, the ship be at rest for a period of two or three hours, the sun may be used at two different times in exactly the same manner as the two stars were used and the problem worked in a manner exactly like the former example. While the sun might be used in this way on land, at sea, however, such a case would be extremely rare, for vessels are seldom or never at rest; they are usually in rapid motion, and during the interval between observations a ship will have travelled many miles.

If the course of the ship and the distance travelled during the interval between the observations be known, then the first Sumner line may be carried forward parallel to itself and drawn through the position of the ship at the time of the second observation. The intersection of this transferred line with the second Sumner line will be the true

position of the ship at the time of the second observation. All of this is at once apparent by an inspection of the accompanying figure.

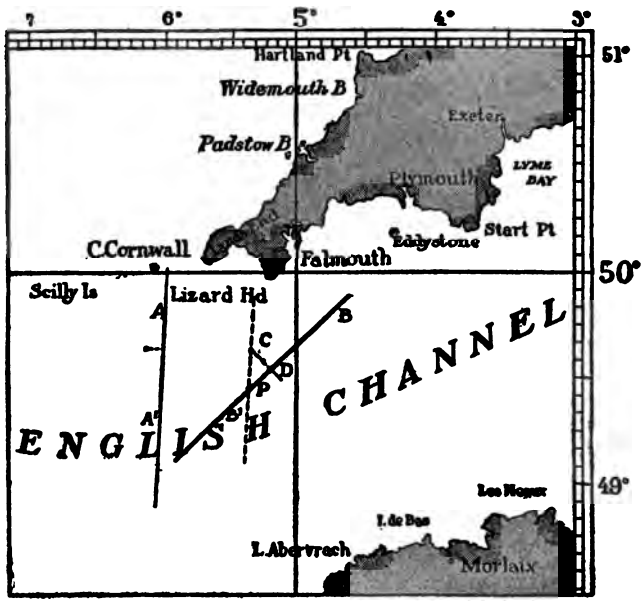


Fig. 15. Lines of Position

At about 8 A. M. a sight was obtained and the Sumner line AA′ drawn on the chart. At 10:30 A. M. a second observation was made. During the interval the ship had run twenty-five miles on a true east course. The approximate position

of the ship at the time of the second sight was, therefore, at C, and through C was drawn a line, CC′, parallel to the first Sumner line. Now it is evident that at 10:30 A. M. the ship must be somewhere on this line. The second sight was worked out and gave a Sumner line BB′ on which the ship must also be. Hence the true position of the vessel at the instant of the second observation was at P, the intersection of the two lines.

If the navigator relied upon the first observation only, he would, by dead reckoning, believe himself to be at C. As a result of the second sight and considering only the results of that sight, he would believe himself at D. The combination of the two sights revealed his true location at P. The original "D. R." position, with which he began the day, was seven (7′) in error in longitude and nine (9′) in latitude.

The latitude and longitude of the point, P, of intersection of the Sumner lines can be easily calculated, without plotting on the chart. The method by which this is done is shown in the accompanying diagram; in which CC′ is the first Sumner line carried forward for the run, C being

the position by dead reckoning at the time of the second sight. DD′ is the second Sumner line, intersecting the first at the point, P, the true posi-

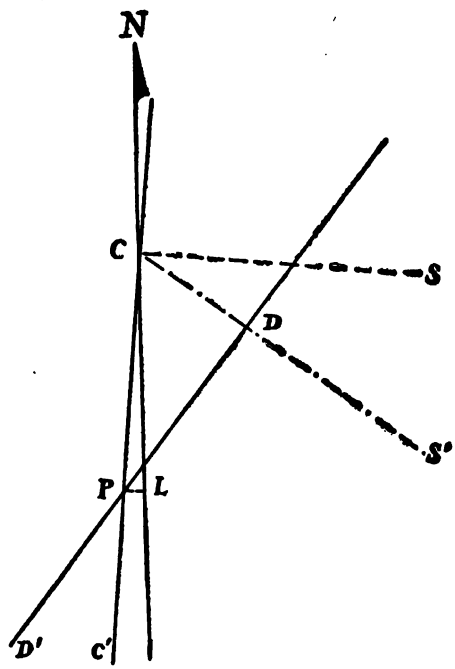


Fig. 16. Finding the Intersection of Lines of Position

tion of the ship: NCS is the azimuth of the sun at the time of the first observation, and NCS′ the azimuth at the second sight.

Now the triangle, CDP, is right-angled at D

and the angle CPD is equal to the difference of the azimuths. The side CD is the distance factor (difference of the altitudes) found in reducing the second sight. Entering the Traverse Tables with this angle CPD as a course and CD as the departure, the side CP, representing the distance, can be at once found. Then this distance CP may be considered as a course sailed by the vessel, and the corresponding difference of latitude, CL, and the departure, PL, can be taken from the Traverse Tables, using CP as the distance and the direction of the Sumner line, CP (azimuth of the sun plus or minus 90°), as the course. After converting this departure into difference of longitude, the differences of latitude and longitude, thus found, can be applied directly to the "D. R." position, C, and hence the latitude and longitude of P, the true position of the ship, found, without drawing either Sumner line and without reference to the chart.

This method of calculation is shown in the following example, which is illustrated in Figure 15. The position by dead reckoning at the time of the second observation, C, was

Latitude 49° 39.5′ N.
Longitude 5° 19′ E.,

at which time the observed altitude of the sun was 58° 9′.

The altitude of the sun as calculated by the St. Hilaire method was 58° 3′, and the corresponding azimuth was N. 136° E. Hence the distance factor, or CD, is 6′, and the difference of the two azimuths (first and second sights), or the angle CPD, is 41°. Entering the Traverse Table with 41° as a course and 6′ as departure, one finds the corresponding distance as 9′. Then entering again the Traverse Tables with direction of the first Sumner line, 185°, as a course and 9′ as distance, one finds:

Difference of Latitude, 9′
Departure, 0.8′

The corresponding difference of longitude is 1.2′, and hence, applying these differences to the "D. R." latitude and longitude, one finds for the true position of the ship at the time of the second observation,

Latitude 49° 30.5′
Longitude 5° 20.2′

and this position agrees with that found by plotting the two Sumner lines on the chart.

CHAPTER V

NOTES AND PRACTICAL APPLICATIONS WITH SPECIAL REFERENCE TO AERIAL NAVIGATION

THE seas have been navigated for centuries and there is, therefore, an immense mass of data available for testing new theories and new methods. The details of observations, taken under favorable and unfavorable conditions, are reported in standard publications and in textbooks, and these may be reduced over and over again by various methods. In this way the relative merits of the various methods may be compared; simplifications discovered and thoroughly tested.

On the other hand the navigation of the air has but begun, and there are no observations, no records, by which theories and methods can be tested. A careful search through the archives of the Aero Club fails to show the record of even one sextant

observation for the determination of the position of an air craft. Such observations may have been made by some Naval aviator, but, if so, no record has been published; not even a note stating the fact that such observations have been made. For this reason the proposed methods of aerial navigation can be but tentative; the fundamental principles can be set forth, but future practice may change many of the details.

On one point practical aviators seem to disagree, and that is, the greatest height at which the actual sea horizon appears clear and distinct. At short distances above sea level, it is sharp and clear, probably even more distinct than from the deck of a vessel; as the height increases, however, the distance which one can see rapidly increases, but the line of demarkation between sea and sky becomes more and more indistinct. At what height this blending of sea and sky becomes so great as to prevent the use of the actual horizon in measuring the altitude of the sun is extremely uncertain and depends largely upon the weather. It seems to be the consensus of opinion, however, that under the most favorable conditions, it is

impossible to clearly distinguish between sea and sky at heights much over two thousand (2000) feet. In other words, sextant observations may be made with some degree of accuracy at heights below two thousand feet but above that height the actual sea horizon cannot be used.

This apparently necessitates an air craft descending to a low height whenever an observation is to be made. This has disadvantages, for time is lost in descending, and further, on account of the dip of the horizon, the height of the air craft above the sea level must be known with considerable accuracy. An error of a couple of hundred feet in the height of the plane as determined by the barograph might make a considerable error in the calculated position of the plane.

It is extremely unfortunate that a practical artificial horizon has not yet been evolved. For with such an instrument an observation could be made at any height and the dip would be completely eliminated. Many attempts have been made to build such an instrument, for it would be useful at sea as well as in the air. But, up to the present time, all such attempts have failed.

Some have appeared successful until subjected to the rigid test of actual practice at sea.

Admiral Fiske has recently suggested that in long distance ocean flights air craft should approximate as closely as possible to the conditions under which marine vessels are navigated; that is, that the air craft should fly as near sea level as possible, at approximately one hundred (100) feet above the surface of the waters. At this height there would be no difference between the navigation of an air craft and that of a marine vessel.

1. The Sight, or Measured Altitude.

At heights under two thousand feet (2000) above the surface of the earth, the altitude of the lower limb of the sun above the true horizon may be directly measured with the sextant. At the moment the altitude is measured, the time must be noted on the chronometer.

The best way to make the measurement is to set the vernier of the sextant to an even reading slightly larger than the altitude of the sun, if the sun be rising (slightly smaller, if the sun be set-

ting), and then watch the edge of the sun and the horizon come together and note on the chronometer the instant of contact.

At the same time note on the barograph the height of the plane above the surface of the earth. With this height enter TABLE III and find the correction to be applied to the measured altitude. This correction is always to be subtracted, so that the true altitude to be used in the calculation will always be less than the reading of the sextant.

If a star, or planet, be observed at night, proceed in exactly the same way.

At heights greater than two thousand feet (2000) the natural horizon cannot be used. In such cases reliance must be placed on an artificial horizon. The measurement is made in the same way as before, but the angle, as read from the sextant, is twice the altitude of the body and the "dip" is now eliminated. The sextant angle is, therefore, to be divided by two (2), and the correction to be applied to this altitude is taken from TABLE II. In using this table note that the correction, when the sun is observed, is always to be

added; while that, when a star is observed, it is always to be subtracted.

2. Finding the Data.

The data consist of the latitude and longitude of the assumed position of the plane at the moment of observation, and the position in the heavens of the observed body (sun, or star).

a. The supposed position of the plane is marked on the chart and the latitude and longitude of this mark is read directly from the scales at the sides of the chart.

This assumed, or D. R., position can, within limits, be so taken as to greatly facilitate the computation. A longitude can be taken, such that, when it is combined with the observed time to find the hour angle, seconds will be eliminated, and the hour angle used in the computation will be a whole number of minutes. As the scale of the mechanical computer is divided to single minutes, settings can be made, when the longitude is so assumed, directly without the estimation of fractions. Again the latitude can be so assumed

as to make L–D a multiple of ten (10′) minutes of arc. The corresponding scale of the Computer is subdivided to 10′, and by means of the assumption, therefore, settings can be made directly.

b. *For the sun.* If the sun is observed, the declination of the sun and the equation of time must be taken from the "Nautical Almanac." These quantities change rapidly and the almanac gives their respective values for Greenwich noon of every day of the year. The values should be taken for the hour nearest the time at which the observation was made.

Correct the observed chronometer time for chronometer error, if any, and for the equation of time, thus finding the Greenwich apparent time. Subtract from this the D. R. longitude (expressed in time) and thus get the Local apparent time, or Ship's time. If the sight be made in the afternoon, this apparent time is the hour angle of the sun, or the t, that is wanted; if the sight be made in the morning, then the local apparent time, as above found, must be sub-

tracted from twelve (12) hours in order to find the hour angle, t.

One thus obtains the data:

Declination of the sun,	D
Latitude of the plane,	L
Hour angle,	t

c. *For a star.* The right ascension and declination of the star are taken from TABLE IV, or from the "Nautical Almanac."

When a sidereal chronometer is used, the observed time is corrected for chronometer error, if any, and the Greenwich sidereal time thus found. From this subtract the D. R. longitude (expressed in time) and thus find the local sidereal time. If this be greater than the right ascension of the star, subtract the right ascension and the result is the west hour angle; if the right ascension be the greater, then subtract from it the sidereal time, and the result will be the east hour angle of the star.

If an ordinary chronometer be used, then the Greenwich mean time as found from it must be converted into Sidereal time by means of the

tables for such conversion given in the "Nautical Almanac." After this, proceed as above.

One thus obtains the data:

Declination of the star,	D
Latitude of the plane,	L
Hour angle,	t

3. Computing the Altitude and Azimuth.

With the data, as above found, the altitude and azimuth of the body are computed by means of the mechanical navigator as fully explained in Chapter II. Only one figure needs be written down and the whole computation can be made in a few moments.

4. Plotting the True Position of the Plane.

From the assumed position, as plotted on the chart, draw a line in the direction of the azimuth of the body as found in 3. On this line, lay off from the D. R. position a distance equal to the difference between the corrected observed altitude, as found in 1, and the computed altitude, as found in 3. If the computed altitude be less than the observed, then this distance is laid off in

the direction of the observed body; if greater, then in the opposite direction.

Through the point thus found draw the Sumner line at right angles to the azimuth line. The plane is somewhere on this line.

5. Practical Examples.

1. In flight July 28, 1918, in approximate latitude 48° N. and longitude 49° W., the altitude of the lower limb of the sun was measured at $9^h\ 14^m\ 20^s$ Greenwich mean time and found to be 15° 27′. The barograph showed the height of the plane to be 2000 feet. Required the line of position?

From the "Nautical Almanac" one finds:

Declination of sun	19° 2′ N.
Equation of time	$6^m\ 20^s$

The hour angle of the sun is found as follows:

Greenwich mean time	9^h	14^m	20^s
Equation of time	—	6	20
Greenwich apparent time	9^h	8^m	0^s
Longitude in time	3	16	0
Hour angle	5^h	52^m	0^s

Whence the data:

Declination, D,	19° 2′ N.
Latitude, L,	48° 0′ N.
L ± D	28° 58′
Hour angle, t,	5^h 52^m W.

With the "Line of Position Computer" the altitude and azimuth are found to be:

Number	305½
Altitude	15° 16′
Azimuth	N 77° W.

To find the correction to be applied to the measured altitude of the sun enter Table III, and the work proceeds as:

Observed altitude	15° 27′
Correction	— 31.5
Corrected altitude	14° 55.5′
Computed altitude	15° 16′
Difference	20.5′

As the observed altitude is smaller than the calculated, the true place of the plane is farther from the sub-solar point, than the assumed place, hence the true place of the plane is 20.5 nautical miles S 77° E. of the approximate place.

To find this true place in the chart, mark the approximate position, (Latitude 48° N. and longitude 49° W.), and from this point lay off a distance of 20½ miles to the direction, S 77° E., and the point thus found will be the most probable position of the plane at the time of making the observation.

2. In flight, July 29, 1918, in approximate latitude 51° 30′ and longitude 12° 30′ W., the altitude of the lower limb of the sun was measured at $5^h\ 10^m$ Greenwich mean time and found to be 31° 50′. The barograph showed the height of the plane to be 1500 feet. Required the most probable position of the plane?

From the "Nautical Almanac" one finds:

Declination of the sun	18° 50′ N.
Equation of time	$6^m\ 19^s$

The hour angle of the sun is found as follows:

Greenwich mean time	$5^h\ 10^m\ 0^s$
Equation of time	— 6 19
Greenwich apparent time	$5^h\ 3^m\ 41^s$
Longitude in time	0 50 0
Hour angle	$4^h\ 13^m\ 41^s$

Whence the data:

Declination, D,	18° 50′ N.
Latitude, L,	51° 30′ N.
L ± D,	32° 40′
Hour angle, t,	$4^h\ 13^m\ 41^s$ W.

And the computation proceeds as:

Number	163
Altitude	31° 0′
Azimuth	N. 99° W.

The observed altitude is now corrected by aid of Table III:

Measured altitude	31° 50′
Correction	— 26
Corrected altitude	31° 24′
Computed altitude	31° 0
Difference	24′

As the observed altitude is larger than the calculated, the true place of the plane is nearer the sub-solar point than the assumed place; hence the true, or rather, the most probable, place of the plane is 24 nautical miles N. 99° W. of the approximate place.

To plot this place in the chart, mark the approximate position by dead reckoning, namely latitude 51° 30′ N. and longitude 12° 30′ W. and from this point lay off in the direction N. 99° W., a distance of 24 miles, and the point thus found will be the most probable position of the plane at the moment of the observation.

TABLE I

Dip and Distance of the Horizon for the Average Condition of the Atmosphere

TABLE I—DIP AND DISTANCE OF HORIZON FOR THE AVERAGE STATE OF THE ATMOSPHERE

To be subtracted from the observed altitude.

Height of eye	Dip.	Distance	Height of eye	Dip.	Distance
	′			′	
0	0.0	0.00	40	6.2	7.26
1	1.0	1.15	41	6.3	7.35
2	1.4	1.62	42	6.4	7.44
3	1.7	1.99	43	6.4	7.53
4	2.0	2.30	44	6.5	7.62
5	2.2	2.57	45	6.6	7.70
6	2.4	2.81	46	6.7	7.79
7	2.6	3.04	47	6.7	7.87
8	2.8	3.25	48	6.8	7.95
9	3.0	3.44	49	6.9	8.04
	′			′	
10	3.1	3.63	50	7.0	8.12
11	3.3	3.81	51	7.0	8.20
12	3.4	3.98	52	7.1	8.28
13	3.6	4.14	53	7.2	8.36
14	3.7	4.30	54	7.2	8.44
15	3.8	4.45	55	7.3	8.52
16	3.9	4.59	56	7.4	8.60
17	4.1	4.73	57	7.5	8.67
18	4.2	4.87	58	7.5	8.74
19	4.3	5.00	59	7.6	8.82
	′			′	
20	4.4	5.13	60	7.6	8.89
21	4.5	5.26	65	7.9	9.26
22	4.6	5.39	70	8.2	9.61
23	4.7	5.51	75	8.5	9.94
24	4.8	5.62	80	8.8	10.27
25	4.9	5.74	85	9.1	10.59
26	5.0	5.85	90	9.3	10.89
27	5.1	5.97	95	9.6	11.19
28	5.2	6.08	100	9.8	11.5
29	5.3	6.18	120	10.8	12.6
	′			′	
30	5.4	6.29	140	11.6	13.6
31	5.5	6.39	160	12.5	14.5
32	5.6	6.50	180	13.2	15.4
33	5.7	6.60	200	13.9	16.2
34	5.7	6.69	300	17.0	19.9
35	5.8	6.79	400	19.7	23.0
36	5.9	6.89	500	22.0	25.7
37	6.0	6.98	600	24.1	28.1
38	6.0	7.08	...	...	...
39	6.1	7.17	...	...	...
	′				
40	6.2	7.26	...	...	...

Height of eye in feet above sea level. Dip in minutes of arc (′). Distance in nautical miles.

TABLE II

CORRECTION FOR REFRACTION, DIP, PARALLAX AND SEMI-DIAMETER, TO BE APPLIED TO OBSERVED ALTITUDES OF THE SUN'S LOWER LIMB, AND THE CORRECTION FOR REFRACTION AND DIP TO BE APPLIED TO THE OBSERVED ALTITUDES OF A STAR (MARINE TABLE)

TABLE II—MARINE TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitude.

Observed Altitude	Height of the Eye							
	0 feet		10 feet		20 feet		30 feet	
	Sun +	Star —	Sun +	Star —	Sun +	Star —	Sun +	Star —
° ′	′	′	′	′	′	′	′	′
6 30	8.2	8.0	5.1	11.1	3.8	12.4	2.8	13.4
7 0	8.7	7.4	5.6	10.5	4.3	11.8	3.3	12.8
30	9.2	7.0	6.1	10.1	4.8	11.4	3.8	12.4
8 0	9.6	6.6	6.5	9.7	5.2	11.0	4.2	12.0
30	9.9	6.2	6.8	9.3	5.5	10.6	4.5	11.6
9 0	10.2	6.0	7.1	9.1	5.8	10.4	4.8	11.4
30	10.6	5.6	7.5	8.7	6.2	10.0	5.2	11.0
10 0	10.8	5.3	7.7	8.4	6.4	9.7	5.4	10.7
30	11.0	5.1	7.9	8.2	6.6	9.5	5.6	10.5
11 0	11.3	4.9	8.2	8.0	6.9	9.3	5.9	10.3
30	11.5	4.7	8.4	7.8	7.1	9.1	6.1	10.1
12 0	11.7	4.5	8.6	7.6	7.3	8.9	6.3	9.9
30	11.8	4.3	8.7	7.4	7.4	8.7	6.4	9.7
13 0	12.0	4.1	8.9	7.2	7.6	8.5	6.6	9.5
30	12.2	4.0	9.1	7.1	7.8	8.4	6.8	9.4
14 0	12.3	3.8	9.2	6.9	7.9	8.2	6.9	9.2
30	12.4	3.7	9.3	6.8	8.0	8.1	7.0	9.1
15 00	12.5	3.6	9.4	6.7	8.1	8.0	7.1	9.0
16	12.8	3.3	9.7	6.4	8.4	7.7	7.4	8.7
17	13.0	3.1	9.9	6.2	8.6	7.5	7.6	8.5
18	13.2	3.0	10.1	6.1	8.8	7.4	7.8	8.4
19	13.3	2.8	10.2	5.9	8.9	7.2	7.9	8.2
20 00	13.5	2.6	10.4	5.7	9.1	7.0	8.1	8.0
22	13.7	2.4	10.6	5.5	9.3	6.8	8.3	7.8
24	13.9	2.2	10.8	5.3	9.5	6.6	8.5	7.6
26	14.2	2.0	11.1	5.1	9.8	6.4	8.8	7.4
28	14.3	1.8	11.2	4.9	9.9	6.2	8.9	7.2
30 00	14.4	1.7	11.3	4.8	10.0	6.1	9.0	7.1
32	14.6	1.5	11.5	4.6	10.2	5.9	9.2	6.9
34	14.7	1.4	11.6	4.5	10.3	5.8	9.3	6.8
36	14.8	1.3	11.7	4.4	10.4	5.7	9.4	6.7
38	14.9	1.2	11.8	4.3	10.5	5.6	9.5	6.6
40 00	15.0	1.2	11.9	4.3	10.6	5.6	9.6	6.6
45	15.1	1.0	12.0	4.1	10.7	5.4	9.7	6.4
50	15.3	0.8	12.2	3.9	10.9	5.2	9.9	6.2
55	15.4	0.7	12.3	3.8	11.0	5.1	10.0	6.1
60	15.5	0.6	12.4	3.7	11.1	5.0	10.1	6.0
65	15.6	0.4	12.5	3.5	11.2	4.8	10.2	5.8
70 00	15.7	0.3	12.6	3.4	11.3	4.7	10.3	5.7
75	15.8	0.2	12.7	3.3	11.4	4.6	10.4	5.6
80	15.8	0.2	12.7	3.3	11.4	4.6	10.4	5.6
85	15.9	0.1	12.8	3.2	11.5	4.5	10.5	5.4
° ′ 90 00	′ 16.0	′ 0.0	′ 12.9	′ 3.1	′ 11.6	′ 4.4	′ 10.6	′ 5.4

Correction for the sun to be *added* to observed altitude.
Correction for a star to be *subtracted* from the observed altitude.

TABLE II—MARINE TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitude.

Observed Altitude	Height of the Eye							
	40 feet		50 feet		60 feet		70 feet	
	Sun +	Star −	Sun +	Star −	Sun +	Star −	Sun +	Star −
° ′	′	′	′	′	′	′	′	′
6 30	2.0	14.2	1.2	15.0	0.6	15.6	0.0	16.2
7 0	2.5	13.6	1.7	14.4	1.1	15.0	0.5	15.6
30	3.0	13.2	2.2	14.0	1.6	14.6	1.0	15.2
8 0	3.4	12.8	2.6	13.6	2.0	14.2	1.4	14.8
30	3.7	12.4	2.9	13.2	2.3	13.8	1.7	14.4
9 0	4.0	12.2	3.2	13.0	2.6	13.6	2.0	14.2
30	4.2	11.8	3.6	12.6	3.0	13.2	2.4	13.8
10 0	4.6	11.5	3.8	12.3	3.2	12.9	2.6	13.5
30	4.8	11.3	4.0	12.1	3.4	12.7	2.8	13.3
11 0	5.1	11.1	4.3	11.9	3.7	12.5	3.1	13.1
30	5.3	10.9	4.5	11.7	3.9	12.3	3.3	12.9
12 0	5.5	10.7	4.7	11.5	4.1	12.1	3.5	12.7
30	5.6	10.5	4.8	11.3	4.2	11.9	3.6	12.5
13 0	5.8	10.3	5.0	11.1	4.4	11.7	3.8	12.3
30	6.0	10.2	5.2	11.0	4.6	11.6	4.0	12.2
14 0	6.1	10.0	5.3	10.8	4.7	11.4	4.1	12.0
30	6.2	9.9	5.4	10.7	4.8	11.3	4.2	11.9
15 00	6.3	9.8	5.5	10.6	4.9	11.2	4.3	11.8
16	6.6	9.5	5.8	10.3	5.2	10.9	4.6	11.5
17	6.8	9.3	6.0	10.1	5.4	10.7	4.8	11.3
18	7.0	9.2	6.2	10.0	5.6	10.6	5.0	11.2
19	7.1	9.0	6.3	9.8	5.7	10.4	5.1	11.0
20 00	7.3	8.8	6.5	9.6	5.9	10.2	5.3	10.8
22	7.5	8.6	6.7	9.4	6.1	10.0	5.5	10.6
24	7.7	8.4	6.9	9.2	6.3	9.8	5.7	10.4
26	8.0	8.2	7.2	9.0	6.6	9.6	6.0	10.2
28	8.1	8.0	7.3	8.8	6.7	9.4	6.1	10.0
30 00	8.2	7.9	7.4	8.7	6.8	9.3	6.2	9.9
32	8.4	7.7	7.6	8.5	7.0	9.1	6.4	9.7
34	8.5	7.6	7.7	8.4	7.1	9.0	6.5	9.6
36	8.6	7.5	7.8	8.3	7.2	8.9	6.6	9.5
38	8.7	7.4	7.9	8.2	7.3	8.8	6.7	9.4
40 00	8.8	7.4	8.0	8.2	7.4	8.8	6.8	9.4
45	8.9	7.2	8.1	8.0	7.5	8.6	6.9	9.2
50	9.1	7.0	8.3	7.8	7.7	8.4	7.1	9.0
55	9.2	6.9	8.4	7.7	7.8	8.3	7.2	8.9
60	9.3	6.8	8.5	7.6	7.9	8.2	7.3	8.8
65	9.4	6.6	8.6	7.4	8.0	8.0	7.4	8.6
70 00	9.5	6.5	8.7	7.3	8.1	7.9	7.5	8.5
75	9.6	6.4	8.8	7.2	8.2	7.8	7.6	8.4
80	9.6	6.4	8.8	7.2	8.2	7.8	7.6	8.4
85	9.7	6.5	8.9	7.1	8.3	7.7	7.7	8.3
° ′	′	′	′	′	′	′	′	′
90 0	9.8	6.2	9.0	7.0	8.4	7.6	7.8	8.2

Correction for the sun to be *added* to observed altitude.
Correction for a star to be *subtracted* from observed altitude.

TABLE II—MARINE TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitude.

Observed Altitude	Height of the Eye							
	0 feet		10 feet		20 feet		30 feet	
	Sun +	Star −	Sun +	Star −	Sun +	Star −	Sun +	Star −
° ′	′	′	′	′	′	′	′	′
6 30	8.2	8.0	5.1	11.1	3.8	12.4	2.8	13.4
7 0	8.7	7.4	5.6	10.5	4.3	11.8	3.3	12.8
30	9.2	7.0	6.1	10.1	4.8	11.4	3.8	12.4
8 0	9.6	6.6	6.5	9.7	5.2	11.0	4.2	12.0
30	9.9	6.2	6.8	9.3	5.5	10.6	4.5	11.6
9 0	10.2	6.0	7.1	9.1	5.8	10.4	4.8	11.4
30	10.6	5.6	7.5	8.7	6.2	10.0	5.2	11.0
10 0	10.8	5.3	7.7	8.4	6.4	9.7	5.4	10.7
30	11.0	5.1	7.9	8.2	6.6	9.5	5.6	10.5
11 0	11.3	4.9	8.2	8.0	6.9	9.3	5.9	10.3
30	11.5	4.7	8.4	7.8	7.1	9.1	6.1	10.1
12 0	11.7	4.5	8.6	7.6	7.3	8.9	6.3	9.9
30	11.8	4.3	8.7	7.4	7.4	8.7	6.4	9.7
13 0	12.0	4.1	8.9	7.2	7.6	8.5	6.6	9.5
30	12.2	4.0	9.1	7.1	7.8	8.4	6.8	9.4
14 0	12.3	3.8	9.2	6.9	7.9	8.2	6.9	9.2
30	12.4	3.7	9.3	6.8	8.0	8.1	7.0	9.1
15 00	12.5	3.6	9.4	6.7	8.1	8.0	7.1	9.0
16	12.8	3.3	9.7	6.4	8.4	7.7	7.4	8.7
17	13.0	3.1	9.9	6.2	8.6	7.5	7.6	8.5
18	13.2	3.0	10.1	6.1	8.8	7.4	7.8	8.4
19	13.3	2.8	10.2	5.9	8.9	7.2	7.9	8.2
20 00	13.5	2.6	10.4	5.7	9.1	7.0	8.1	8.0
22	13.7	2.4	10.6	5.5	9.3	6.8	8.3	7.8
24	13.9	2.2	10.8	5.3	9.5	6.6	8.5	7.6
26	14.2	2.0	11.1	5.1	9.8	6.4	8.8	7.4
28	14.3	1.8	11.2	4.9	9.9	6.2	8.9	7.2
30 00	14.4	1.7	11.3	4.8	10.0	6.1	9.0	7.1
32	14.6	1.5	11.5	4.6	10.2	5.9	9.2	6.9
34	14.7	1.4	11.6	4.5	10.3	5.8	9.3	6.8
36	14.8	1.3	11.7	4.4	10.4	5.7	9.4	6.7
38	14.9	1.2	11.8	4.3	10.5	5.6	9.5	6.6
40 00	15.0	1.2	11.9	4.3	10.6	5.6	9.6	6.6
45	15.1	1.0	12.0	4.1	10.7	5.4	9.7	6.4
50	15.3	0.8	12.2	3.9	10.9	5.2	9.9	6.2
55	15.4	0.7	12.3	3.8	11.0	5.1	10.0	6.1
60	15.5	0.6	12.4	3.7	11.1	5.0	10.1	6.0
65	15.6	0.4	12.5	3.5	11.2	4.8	10.2	5.8
70 00	15.7	0.3	12.6	3.4	11.3	4.7	10.3	5.7
75	15.8	0.2	12.7	3.3	11.4	4.6	10.4	5.6
80	15.8	0.2	12.7	3.3	11.4	4.6	10.4	5.6
85	15.9	0.1	12.8	3.2	11.5	4.5	10.5	5.4
° ′	′	′	′	′	′	′	′	′
90 00	16.0	0.0	12.9	3.1	11.6	4.4	10.6	5.4

Correction for the sun to be *added* to observed altitude.
Correction for a star to be *subtracted* from the observed altitude.

TABLE II—MARINE TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitude.

Observed Altitude	Height of the Eye							
	40 feet		50 feet		60 feet		70 feet	
	Sun +	Star −	Sun +	Star −	Sun +	Star −	Sun +	Star −
° ′	′	′	′	′	′	′	′	′
6 30	2.0	14.2	1.2	15.0	0.6	15.6	0.0	16.2
7 0	2.5	13.6	1.7	14.4	1.1	15.0	0.5	15.6
30	3.0	13.2	2.2	14.0	1.6	14.6	1.0	15.2
8 0	3.4	12.8	2.6	13.6	2.0	14.2	1.4	14.8
30	3.7	12.4	2.9	13.2	2.3	13.8	1.7	14.4
9 0	4.0	12.2	3.2	13.0	2.6	13.6	2.0	14.2
30	4.2	11.8	3.6	12.6	3.0	13.2	2.4	13.8
10 0	4.6	11.5	3.8	12.3	3.2	12.9	2.6	13.5
30	4.8	11.3	4.0	12.1	3.4	12.7	2.8	13.3
11 0	5.1	11.1	4.3	11.9	3.7	12.5	3.1	13.1
30	5.3	10.9	4.5	11.7	3.9	12.3	3.3	12.9
12 0	5.5	10.7	4.7	11.5	4.1	12.1	3.5	12.7
30	5.6	10.5	4.8	11.3	4.2	11.9	3.6	12.5
13 0	5.8	10.3	5.0	11.1	4.4	11.7	3.8	12.3
30	6.0	10.2	5.2	11.0	4.6	11.6	4.0	12.2
14 0	6.1	10.0	5.3	10.8	4.7	11.4	4.1	12.0
30	6.2	9.9	5.4	10.7	4.8	11.3	4.2	11.9
15 00	6.3	9.8	5.5	10.6	4.9	11.2	4.3	11.8
16	6.6	9.5	5.8	10.3	5.2	10.9	4.6	11.5
17	6.8	9.3	6.0	10.1	5.4	10.7	4.8	11.3
18	7.0	9.2	6.2	10.0	5.6	10.6	5.0	11.2
19	7.1	9.0	6.3	9.8	5.7	10.4	5.1	11.0
20 00	7.3	8.8	6.5	9.6	5.9	10.2	5.3	10.8
22	7.5	8.6	6.7	9.4	6.1	10.0	5.5	10.6
24	7.7	8.4	6.9	9.2	6.3	9.8	5.7	10.4
26	8.0	8.2	7.2	9.0	6.6	9.6	6.0	10.2
28	8.1	8.0	7.3	8.8	6.7	9.4	6.1	10.0
30 00	8.2	7.9	7.4	8.7	6.8	9.3	6.2	9.9
32	8.4	7.7	7.6	8.5	7.0	9.1	6.4	9.7
34	8.5	7.6	7.7	8.4	7.1	9.0	6.5	9.6
36	8.6	7.5	7.8	8.3	7.2	8.9	6.6	9.5
38	8.7	7.4	7.9	8.2	7.3	8.8	6.7	9.4
40 00	8.8	7.4	8.0	8.2	7.4	8.8	6.8	9.4
45	8.9	7.2	8.1	8.0	7.5	8.6	6.9	9.2
50	9.1	7.0	8.3	7.8	7.7	8.4	7.1	9.0
55	9.2	6.9	8.4	7.7	7.8	8.3	7.2	8.9
60	9.3	6.8	8.5	7.6	7.9	8.2	7.3	8.8
65	9.4	6.6	8.6	7.4	8.0	8.0	7.4	8.6
70 00	9.5	6.5	8.7	7.3	8.1	7.9	7.5	8.5
75	9.6	6.4	8.8	7.2	8.2	7.8	7.6	8.4
80	9.6	6.4	8.8	7.2	8.2	7.8	7.6	8.4
85	9.7	6.5	8.9	7.1	8.3	7.7	7.7	8.3
° ′	′	′	′	′	′	′	′	′
90 0	9.8	6.2	9.0	7.0	8.4	7.6	7.8	8.2

Correction for the sun to be *added* to observed altitude.
Correction for a star to be *subtracted* from observed altitude.

TABLE II—MARINE TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitude.

Observed Altitude	Height of the Eye							
	80 feet		90 feet		100 feet		120 feet	
	Sun ∓	Star —	Sun ∓	Star —	Sun ∓	Star —	Sun ∓	Star —
° ′	′	′	′	′	′	′	′	′
6 0	−0.6	16.8	−1.1	17.3	−1.6	17.8	−2.6	18.8
7 0	−0.1	16.2	−0.6	16.7	−1.1	17.2	−2.1	18.2
30	+0.4	15.8	−0.1	16.3	−0.6	16.8	−1.6	17.8
8 0	0.8	15.4	+0.3	15.9	−0.2	16.4	−1.2	17.4
30	1.1	15.0	0.6	15.5	+0.1	16.0	−0.9	17.0
9 0	1.4	14.8	0.9	15.3	0.4	15.8	−0.6	16.8
30	1.8	14.4	1.3	14.9	0.8	15.4	−0.2	16.4
10 0	2.0	14.1	1.5	14.6	1.0	15.1	0.0	16.1
30	2.2	13.9	1.7	14.4	1.2	14.9	+0.2	15.9
11 0	2.5	13.7	2.0	14.2	1.5	14.7	0.5	15.7
30	2.7	13.5	2.2	14.0	1.7	14.5	0.7	15.5
12 0	2.9	13.3	2.4	13.8	1.9	14.3	0.9	15.3
30	3.0	13.1	2.5	13.6	2.0	14.1	1.0	15.1
13 0	3.2	12.9	2.7	13.4	2.2	13.9	1.2	14.9
30	3.4	12.8	2.9	13.3	2.4	13.8	1.4	14.8
14 0	3.5	12.6	3.0	13.1	2.5	13.6	1.5	14.6
30	3.6	12.5	3.1	13.0	2.6	13.5	1.6	14.5
15 00	3.7	12.4	3.2	12.9	2.7	13.4	1.7	14.4
16	4.0	12.1	3.5	12.6	3.0	13.1	2.0	14.1
17	4.2	11.9	3.7	12.4	3.2	12.9	2.2	13.9
18	4.4	11.8	3.9	12.3	3.4	12.8	2.4	13.8
19	4.5	11.6	4.0	12.1	3.5	12.6	2.5	13.6
20 00	4.7	11.4	4.2	11.9	3.7	12.4	2.7	13.4
22	4.9	11.2	4.4	11.7	3.9	12.2	2.9	13.2
24	5.1	11.0	4.6	11.5	4.1	12.0	3.0	13.0
26	5.4	10.8	4.9	11.3	4.4	11.8	3.4	12.8
28	5.5	10.6	5.0	11.1	4.5	11.6	3.5	12.6
30 00	5.6	10.5	5.1	11.0	4.6	11.5	3.6	12.5
32	5.8	10.3	5.3	10.8	4.8	11.3	3.8	12.3
34	5.9	10.2	5.4	10.7	4.9	11.2	3.9	12.2
36	6.0	10.1	5.5	10.6	5.0	11.1	4.0	12.1
38	6.1	10.0	5.6	10.5	5.1	11.0	4.1	12.0
40 00	6.2	10.0	5.7	10.5	5.2	11.0	4.2	12.0
45	6.3	9.8	5.8	10.3	5.3	10.8	4.3	11.8
50	6.5	9.6	6.0	10.1	5.5	10.6	4.5	11.6
55	6.6	9.5	6.1	10.0	5.6	10.5	4.6	11.5
60	6.7	9.4	6.2	9.9	5.7	10.4	4.7	11.4
65	6.8	9.2	6.3	9.7	5.8	10.2	4.8	11.2
70 00	6.9	9.1	6.4	9.6	5.9	10.1	4.9	11.1
75	7.0	9.0	6.5	9.5	6.0	10.0	5.0	11.0
80	7.0	9.0	6.5	9.5	6.0	10.0	5.0	11.0
85	7.1	8.9	6.6	9.4	6.1	9.9	5.1	10.9
° ′	′	′	′	′	′	′	′	′
90 00	+7.2	8.8	+6.7	9.3	+6.2	9.8	+5.2	10.8

Correction for a star to be subtracted from observed altitude.
observed altitude in accordance with sign.
Correction for a star to be subtracted from observed altitude.

TABLE III

CORRECTION FOR REFRACTION, DIP, PARALLAX AND SEMI-DIAMETER, TO BE APPLIED TO OBSERVED ALTITUDES OF THE SUN'S LOWER LIMB, AND THE CORRECTION FOR REFRACTION AND DIP TO BE APPLIED TO THE OBSERVED ALTITUDES OF A STAR (AERIAL TABLE)

TABLE III—AERIAL TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitudes of the Sun.

Observed Altitude	Height of Air-craft in feet					
	0	100	200	300	400	500
° ′	′	′	′	′	′	′
6 30	+ 8	—2	—6	—9	—12	—14
7 0	9	—1	5	8	11	13
30	9	—1	5	8	11	13
8 0	10	0	4	7	10	12
30	10	0	4	7	10	12
9 0	10	0	4	7	10	12
30	11	+1	3	6	9	11
10 0	11	1	3	6	9	11
30	11	1	3	6	9	11
11 0	11	1	3	6	9	11
30	12	2	2	5	8	10
12 0	12	2	2	5	8	10
30	12	2	2	5	8	10
13 0	12	2	2	5	8	10
30	12	2	2	5	8	10
14 0	12	2	2	5	8	10
30	12	3	2	5	8	10
15 00	13	3	—1	4	7	9
16	13	3	1	4	7	9
17	13	3	1	4	7	9
18	13	3	1	4	7	9
19	13	4	—1	4	7	9
20 00	14	4	0	3	6	8
22	14	4	0	3	6	8
24	14	4	0	3	6	8
26	14	4	0	3	6	8
28	14	4	0	3	6	8
30 00	14	5	0	3	6	8
32	15	5	+1	2	5	7
34	15	5	1	2	5	7
36	15	5	1	2	5	7
38	15	5	1	2	5	7
40 00	15	5	1	2	5	7
45	15	5	1	2	5	7
50	15	5	1	2	5	7
55	15	6	1	2	5	7
60	16	6	2	1	4	6
65	16	6	2	1	4	6
70 00	16	6	2	1	4	6
75	16	6	2	1	4	6
80	16	6	2	1	4	6
85	16	6	2	1	4	6
° ′	′	′	′	′	′	′
90 00	+16	+6	+2	—1	—4	—6

Below 300 feet the correction to be added to, or subtracted from observed altitude, as indicated by sign.

Above 300 feet the correction is always to be subtracted from observed altitude.

TABLE III—AERIAL TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitudes of the Sun.

Observed Altitude	Height of Air-craft in feet					
	750	1000	1250	1500	1750	2000
° ′	′	′	′	′	′	′
6 30	—19	—23	—27	—30	—33	—36
7 0	18	22	26	29	32	35
30	18	22	26	29	32	35
8 0	17	21	25	28	31	34
30	17	21	25	28	31	34
9 0	17	21	25	28	31	34
30	16	20	24	27	30	33
10 0	16	20	24	27	30	33
30	16	20	24	27	30	33
11 0	16	20	24	27	30	33
30	15	19	23	26	29	32
12 0	15	19	23	26	29	32
30	15	19	23	26	29	32
13 0	15	19	23	26	29	32
30	15	19	23	26	29	32
14 0	15	19	23	26	29	32
30	15	19	23	26	29	32
15 00	14	18	22	25	28	31
16	14	18	22	25	28	31
17	14	18	22	25	28	31
18	14	18	22	25	28	31
19	14	18	22	25	28	31
20 00	13	17	21	24	27	30
22	13	17	21	24	27	30
24	13	17	21	24	27	30
26	13	17	21	24	27	30
28	13	17	21	24	27	30
30 00	13	17	21	24	27	30
32	12	16	20	23	26	29
34	12	16	20	23	26	29
36	12	16	20	23	26	29
38	12	16	20	23	26	29
40 00	12	16	20	23	26	29
45	12	16	20	23	26	29
50	12	16	20	23	26	29
55	12	16	20	23	26	29
60	11	15	19	22	25	28
65	11	15	19	22	25	28
70 00	11	15	19	22	25	28
75	11	15	19	22	25	28
80	11	15	19	22	25	28
85	11	15	19	22	25	28
° ′ 90 00	′ —11	′ —15	′ —19	′ —22	′ —25	′ —28

Correction to be subtracted from the observed altitude.

TABLE III—AERIAL TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitudes of a Star.

Observed Altitude	Height of Air-craft in feet					
	0	100	200	300	400	500
° ′	′	′	′	′	′	′
6 30	8	18	22	25	28	30
7 0	7	17	21	24	27	29
30	7	17	21	24	27	29
8 0	6	16	20	23	26	28
30	6	16	20	23	26	28
9 0	6	16	20	23	26	28
30	6	16	20	23	26	28
10 0	5	15	19	22	25	27
30	5	15	19	22	25	27
11 0	5	15	19	22	25	27
30	5	15	19	22	25	27
12 0	4	14	18	21	24	26
30	4	14	18	21	24	26
13 0	4	14	18	21	24	26
30	4	14	18	21	24	26
14 0	4	14	18	21	24	26
30	4	14	18	21	24	26
15 00	4	14	18	21	24	26
16	3	13	17	20	23	25
17	3	13	17	20	23	25
18	3	13	17	20	23	25
19	3	13	17	20	23	25
20 00	3	13	17	20	23	25
22	2	12	16	19	22	24
24	2	12	16	19	22	24
26	2	12	16	19	22	24
28	2	12	16	19	22	24
30 00	2	12	16	19	22	24
32	1	11	15	18	21	23
34	1	11	15	18	21	23
36	1	11	15	18	21	23
38	1	11	15	18	21	23
40 00	1	11	15	18	21	23
45	1	11	15	18	21	23
50	1	11	15	18	21	23
55	1	11	15	18	21	23
60	1	11	15	18	21	23
65	0	10	14	17	20	22
70 00	0	10	14	17	20	22
75	0	10	14	17	20	22
80	0	10	14	17	20	22
85	0	10	14	17	20	22
° ′	′	′	′	′	′	′
90 00	0	10	14	17	20	22

Correction to be *subtracted* from observed altitude.

TABLE III—AERIAL TABLE FOR THE AVERAGE STATE OF THE ATMOSPHERE

Correction to be applied to observed altitudes of a Star.

Observed Altitude	Height of Air-craft in feet					
	750	1000	1250	1500	1750	2000
° ′	′	′	′	′	′	′
6 30	35	39	43	46	49	52
7 0	34	38	42	45	48	51
30	34	38	42	45	48	51
8 0	33	37	41	44	47	50
30	33	37	41	44	47	50
9 0	33	37	41	44	47	50
30	33	37	41	44	47	50
10 0	32	36	40	43	46	49
30	32	36	40	43	46	49
11 0	32	36	40	43	46	49
30	32	36	40	43	46	49
12 0	31	35	39	42	45	48
30	31	35	39	42	45	48
13 0	31	35	39	42	45	48
30	31	35	39	42	45	48
14 0	31	35	39	42	45	48
30	31	35	39	42	45	48
15 00	31	35	39	42	45	48
16	30	34	38	41	44	47
17	30	34	38	41	44	47
18	30	34	38	41	44	47
19	30	34	38	41	44	47
20 00	30	34	38	41	44	47
22	29	33	37	40	43	46
24	29	33	37	40	43	46
26	29	33	37	40	43	46
28	29	33	37	40	43	46
30 00	29	33	37	40	43	46
32	28	32	36	39	42	45
34	28	32	36	39	42	45
36	28	32	36	39	42	45
38	28	32	36	39	42	45
40 00	28	32	36	39	42	45
45	28	32	36	39	42	45
50	28	32	36	39	42	45
55	28	32	36	39	42	45
60	28	32	36	39	42	45
65	27	31	35	38	41	44
70 00	27	31	35	38	41	44
75	27	31	35	38	41	44
80	27	31	35	38	41	44
85	27	31	35	38	41	44
° ′	′	′	′	′	′	′
90 00	27	31	35	38	41	44

Correction to be *subtracted* from observed altitude.

TABLE IV

Fifteen Useful Navigational Stars
Epoch 1918

TABLE IV—FIFTEEN USEFUL NAVIGATIONAL STARS
EPOCH 1918

Mag.	Name		Right ascension	Declination	No.
			h m s	° ′	
2.2	α Andromeda	Alpheratz	0 4 9	+28 38.3	
1.9	α Persei	Mirfac	3 18 28	+49 34.2	11
0.2	α Aurigea	Capella	5 10 38	+45 54.9	10
—0.3	β Orionis	Rigel	5 10 36	— 8 17.7	
—0.9	α Argus	Canopus	6 22 8	—52 39.0	13
—1.6	α Canis Majoris	Sirius	6 41 32	—16 36.2	
2.0	α^2 Geminorum	Castor	7 29 22	+32 4.2	7
1.3	α Leonis	Regulus	10 4 0	+12 22.1	
2.0	α Ursa Majoris	Dubhe	10 58 41	+62 11.6	15
1.9	η Ursa Majoris	Alkaid	13 44 19	+49 43.3	12
0.2	α Boötis	Arcturus	14 11 55	+19 36.5	
1.2	α Scorpii	Antares	16 24 23	—26 15.1	
0.1	α Lyrae	Vega	18 34 10	+38 42.4	8
0.9	α Aquilae	Altair	19 46 47	+ 8 39.1	
1.3	α Cygni	Deneb	20 38 38	+44 59.2	9